Chapter 3

Basic Geometry

Learning Objectives

After completing this chapter, you will be able to

- know the common terms and notations in geometry, such as points, lines (segments), planes, solids, angles, polygons, regular polygons, polyhedra and regular polyhedra.
- identify different types of angles and polygons.
- know the properties of solids, such as Euler's formula.
- draw 2-D representations of simple solids.
- draw cross-sections of solids.
- use drawing tools, such as rulers, compasses and set squares, to draw polygons, circles, parallel lines and perpendicular lines.

A tangram is formed by cutting a square board into 7 polygons. It can be used to form different shapes. Use a tangram to form the following shapes.

(a) Triangle
(b) Rectangle
(c) Trapezium

[A tangram is provided in the Appendix.]

Preview

[Basic knowledge required for this chapter.]

Basic Knowledge

1. **Points**

It is easy to find points around us.

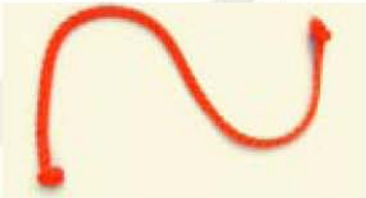
e.g.

2. Lines

Lines are commonly seen in our daily life as well.

e.g. **(a)** The shape of a high jump bar is a **straight line**.

(b) The shape of the string in the picture is a **curve**.



3. **Planes**

In a classroom, the surfaces of blackboard, desks and the floor are all planes.

point 點　straight line 直綫　curve 曲綫　plane 平面

3.1 Points and Lines

Geometry is a branch of Mathematics which studies the properties of figures and spaces. In this chapter, we will learn some basic knowledge in geometry.

A Points

In geometry, a point does not have shape and size. It only shows a location. We usually use a small dot '•' or a small cross '×' to indicate a point, and label it with a capital letter. (See Figure 3.1)

Figure 3.1

B Lines

(a) In geometry, a line has no thickness or breadth, but only length. It can be extended in both directions without end.

Here are two ways to label a straight line.

(i) Label a straight line using a letter on the side.

Figure 3.2

e.g. In Figure 3.2, the straight line is denoted by line ℓ.

(ii) Label two points on a straight line using two capital letters.

Figure 3.3

e.g. In Figure 3.3, the straight line is denoted by line AB (or line BA).

(b) If one end of a line is fixed but the other end can be extended infinitely, the line is called a **ray**.

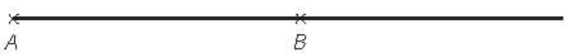


Figure 3.4

e.g. In Figure 3.4, the ray is called ray AB.

geometry 幾何學　　ray 射綫

(c) A **line segment** is a part of a line with two end points.

Figure 3.5

e.g. In Figure 3.5, the line segment is called line segment AB (or line segment BA).

We can use the name of a line segment to denote its length.

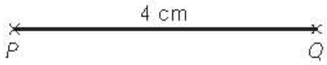


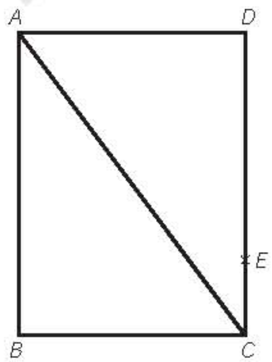
Figure 3.6

e.g. In Figure 3.6, $PQ = 4$ cm.

Extension 3.1

Solution

Measure the line segments in the figure and answer the following questions.



(a) $AC =$ ________ cm

(b) $AB + BC =$ ________ cm + ________ cm

$=$ ________ cm

(c) $DE =$ ________ cm

(d) $CD - CE =$ ________ cm − ________ cm

$=$ ________ cm

3.2 Angles

A Naming angles

Many things in our daily life are related to **angles**. Figure 3.7 shows some examples.

(a) (b) (c)

Figure 3.7

line segment 綫段　　angle 角

In geometry, angles are formed under the following situations:

(a) Two rays from a point (see Figure 3.8(a));

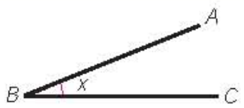
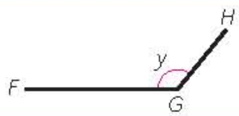
(b) Rotation (see Figure 3.8(b));

(c) Two lines intersecting each other (see Figure 3.8(c)).

(a) (b) (c)

Figure 3.8

There are three ways to name an angle as shown below:

Angle	Method 1	Method 2	Method 3
	$\angle ABC$ or $\angle CBA$	$\angle B$	x
	$\angle FGH$ or $\angle HGF$	$\angle G$	y

Notes: **(a)** '$\angle$' is the symbol of an angle.

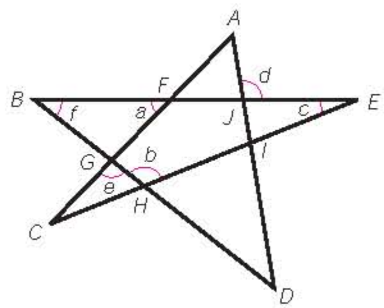
(b) In $\angle ABC$ and $\angle CBA$, point B is the vertex of the angle.

(c) Method 2 can only be used when no confusion will be caused.

Extension 3.2

Solution

Referring to the figure, answer the following questions.



(a) $\angle GHI =$ ____________

(b) $\angle AJE =$ ____________

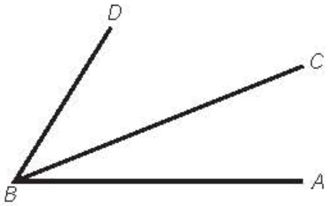
(c) ____________ $= e$

(d) ____________ $= a$

Example 3.1 Naming the angles

Additional Example 3.1

Name all the angles in the following figure.



Here, B is a common vertex for these three angles, so '$\angle B$' cannot indicate which angle it is referring to.

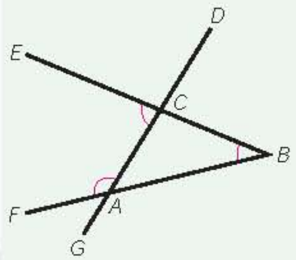
Solution

The three angles are $\underline{\underline{\angle ABC, \angle ABD \text{ and } \angle CBD}}$.

Classwork 3.1

Solution

Name all marked angles in the following figure.



B Unit of angles

When an object rotates 1 turn, it returns to its original position, and the angle formed is called a **round angle**. If we divide a round angle into 360 equal parts, the size of the angle of each part is called one **degree** which is denoted by '1°'.

The angle formed by rotating the arm 1 turn is 360°.

1 round angle = 360°

e.g. Figure 3.9(a), (b) and (c) show angles in $\frac{1}{4}$ of a round angle, $\frac{1}{2}$ of a round angle and $\frac{3}{4}$ of a round angle respectively.

(a) (b) (c)

Figure 3.9

Notes: Sometimes, we need finer units to measure an angle.

(a) 1 **minute** $= \frac{1}{60}$ degree

$\therefore$ 1 degree = 60 minutes (written as 60′)

(b) 1 **second** $= \frac{1}{60}$ minute

$\therefore$ 1 minute = 60 seconds (written as 60″)

Therefore 1° = 60′ = 3 600″

Ancient people thought that the Earth was the centre of the universe. And thus, they thought that the Sun rotated 1 turn around the Earth in one year, and $\frac{1}{365}$ of a turn in one day, i.e. nearly $\frac{1}{360}$ of a turn in one day. This was also the origin of the definition of 1° as $\frac{1}{360}$ of a round angle by ancient *Babylonians*.

round angle 周角　　degree 度
minute 分　　second 秒

Babylonians 巴比倫人

Additional Example 3.2

Example 3.2 Expressing angles in degrees

Express the following angles in degrees.

(a) 2 round angles

(b) $\frac{1}{3}$ of a round angle

Solution

(a) 2 round angles $= 2 \times 360°$ ◂ 1 round angle $= 360°$

$= \underline{\underline{720°}}$

(b) $\frac{1}{3}$ of a round angle $= \frac{1}{3} \times 360°$

$= \underline{\underline{120°}}$

Classwork 3.2

Solution

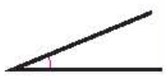
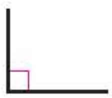
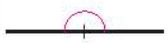
Express the following angles in degrees.

(a) 5 round angles

(b) $\frac{1}{8}$ of a round angle

C Types of angles

The following table shows the types of angles classified by their sizes.

Type	Acute angle	Right angle	Obtuse angle	Straight angle	Reflex angle	Round angle
Diagram						
Size	Between 0° and 90°	90°	Between 90° and 180°	180°	Between 180° and 360°	360°

Note: In Figure 3.10, $\angle ABC$ represents a, reflex angle ABC represents b.

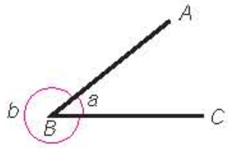


Figure 3.10

I.T. Tool

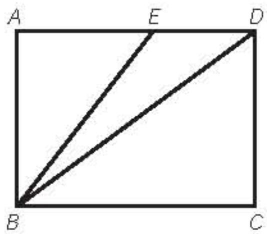
Skills Upgrading Corner 3.1

Solution

In the figure, $ABCD$ is a rectangle.

(a) Write down an acute angle with BD as its arm.

(b) Write down an obtuse angle with E as its vertex.

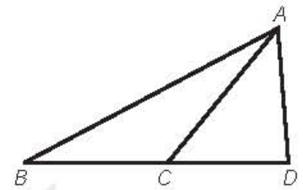


acute angle 銳角 right angle 直角 obtuse angle 鈍角
straight angle 平角 reflex angle 反角

Exercise 3A

Level 1

1. Measure the line segments in the figure and find the lengths of the following expressions.

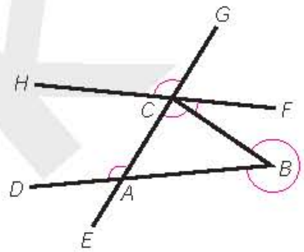


(a) AB

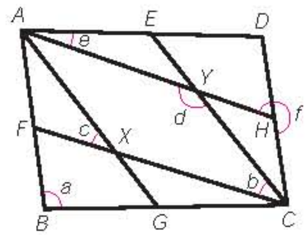
(b) $AC+CB$

(c) $BD-BC$

2. Name all the marked angles in the figure.



3. Referring to the figure, answer the following questions.



(a) $\angle AXF =$ ____________

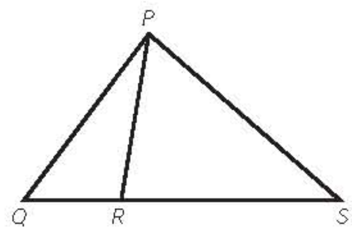
(b) $\angle XCY =$ ____________

(c) ____________ $= e$

(d) ____________ $= f$

4. According to the figure, complete the following table.

Angle	$\angle RPS$	$\angle Q$		
Vertex			R	
Arms			PR, RQ	PR, RS



5. Express the following angles in degrees.

(a) 3 round angles

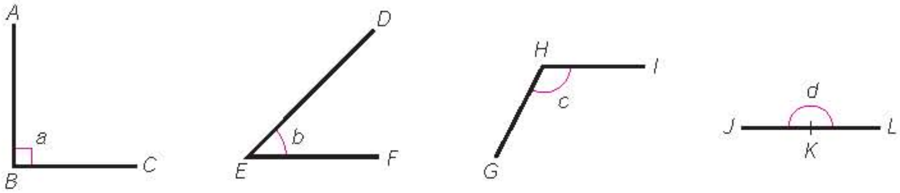
(b) $\frac{1}{4}$ of a round angle

(c) $\frac{5}{9}$ of a round angle

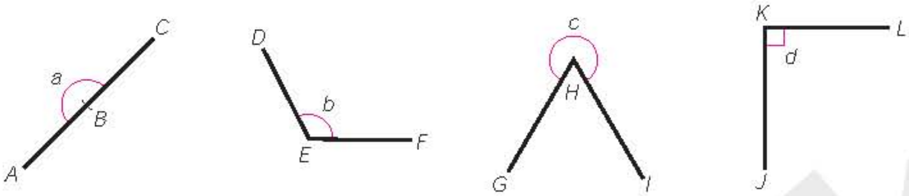
(d) $3\frac{1}{2}$ round angles

6. In each of the following groups of angles, arrange a, b, c and d in descending order of sizes.

(a)



(b)



7. Classify angles a to j below appropriately in the table.

$a = 8°$, $b = 39°$, $c = 150°$, $d = 340°$, $e = 255°$, $f = 86°$, $g = 174°$, $h = 312°$, $i = 99°$, $j = 258°$

Type	Acute angle	Obtuse angle	Reflex angle
Name			

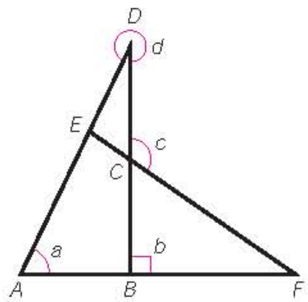
8. For the marked angles in the figure,

(a) which angle is an acute angle?

(b) which angle is a right angle?

(c) which angle is an obtuse angle?

(d) which angle is a reflex angle?



Level 2

9. In each of the following figures, find the marked angle between the hour-hand and the minute-hand.

(a)

(b)

(c)

10. **(a)** When a minute-hand makes 1 turn, how many degrees does the hour-hand turn?

(b) When a minute-hand makes $\frac{1}{4}$ of a turn, how many degrees does the hour-hand turn?

(c) At 12:15, how many degrees is the acute angle between the hour-hand and the minute-hand?

3.3 The Use of Protractor

A **protractor** is a tool for measuring and drawing angles. It is semi-circular in shape with a base line on it. It is divided into 180 equal parts with a **scale** of 1° each. A protractor usually has two scales running in opposite directions from 0° to 180°. Figure 3.11 shows a protractor.

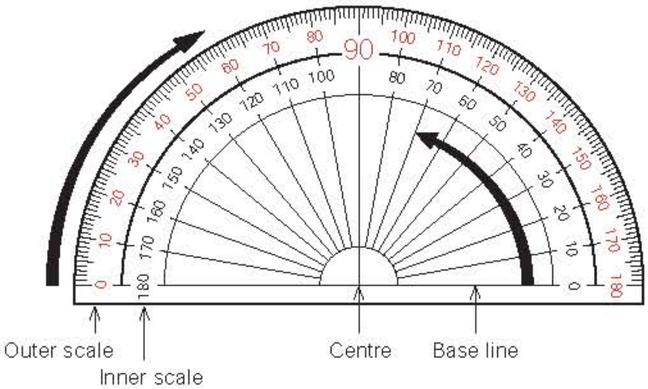


Figure 3.11

A Measuring angles

Table 3.1 shows the steps of measuring acute, right or obtuse angles.

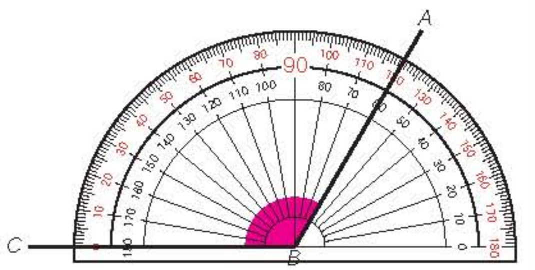
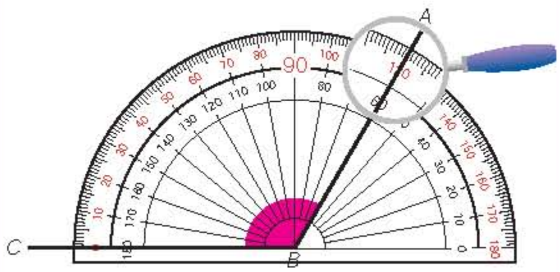
Step 1	Place the protractor with its base line overlapping an arm of the angle (e.g. BC), and its centre lying on the vertex (i.e. point B).	
Step 2	Read the scale with 0° lying on BC. Measure the angle with the outer scale, and get $\angle ABC = 120°$.	

Table 3.1

Attention! The bottom edge of the protractor is not the base line.

Note: You can extend the arms of the angle for measurement if necessary.

I.T. Tool

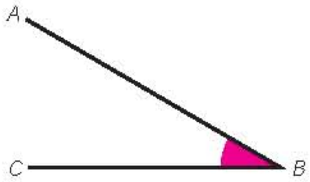
protractor 量角器　　scale 標度

Extension 3.3

Solution

Find the sizes of the following marked angles by using a protractor.

(a)



(b)

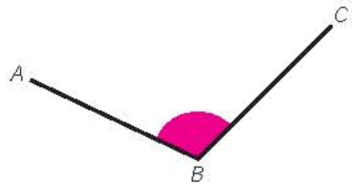


Table 3.2 shows the steps of measuring reflex angles.

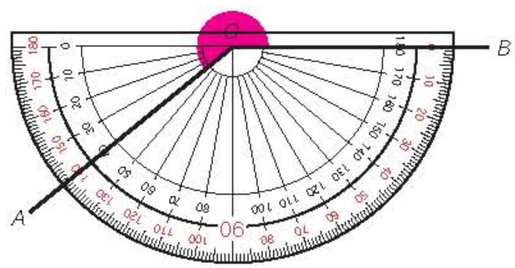
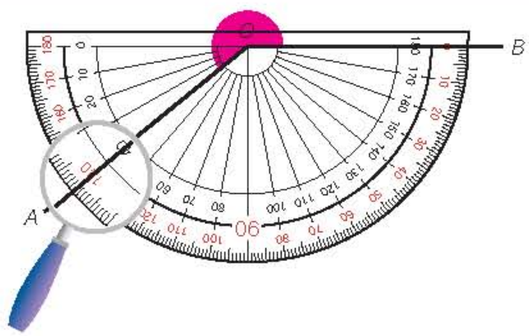
Step 1	Place the protractor upside down with its base line lying on OB, and its centre lying on O.	
Step 2	Read the scale with 0° lying on OB. Measure the angle with the outer scale, and get $\angle AOB = 140°$. $\therefore$ Reflex angle AOB $= 360° - \angle AOB$ $= 360° - 140°$ $= 220°$	

Table 3.2

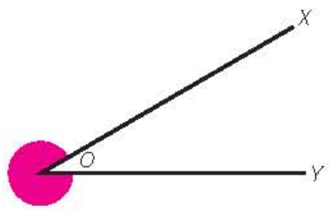
I.T. Tool

Extension 3.4

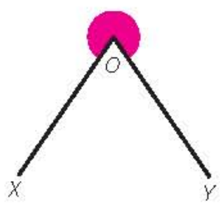
Solution

Find the sizes of the following marked reflex angles with a protractor.

(a)



(b)



B Drawing angles

Table 3.3 shows the steps of drawing acute, right or obtuse angles by taking $\angle AOB = 54°$ as an example.

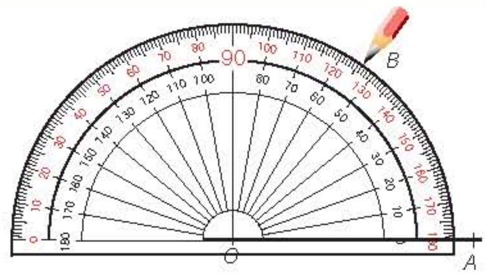
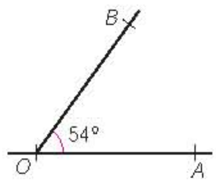
Step 1	Draw a straight line OA.	
Step 2	**(a)** Place the base line of the protractor on OA with the centre at O. **(b)** At the reading 54° from the inner scale (126° from the outer scale), mark point B.	
Step 3	Join OB. Then $\angle AOB = 54°$.	

Table 3.3

Table 3.4 shows the steps of drawing reflex angles by taking reflex angle $POQ = 260°$ as an example.

◂ As $260° = 360° - 100°$, we have to draw an angle of 100° first.

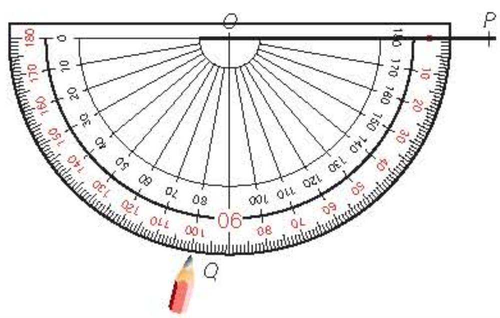
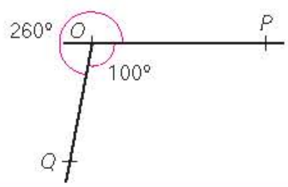
Step 1	Draw a straight line OP.	
Step 2	**(a)** Place the protractor upside down, with its base line lying on OP and its centre at O. **(b)** At the reading 100° from the outer scale, mark point Q.	
Step 3	Join OQ. Then we have $\angle POQ = 100°$. $\therefore$ Reflex angle $POQ = 360° - \angle POQ$ $= 360° - 100°$ $= 260°$	

Table 3.4

Extension 3.5

Solution

Draw the following angles with a protractor.

(a) 64° **(b)** 110° **(c)** 250°

Skills Upgrading Corner 3.2

Solution

(a) In the figure, draw line segments AB and DC, where $\angle ABC = 45°$, $\angle BCD = 100°$, A and D must lie above BC.

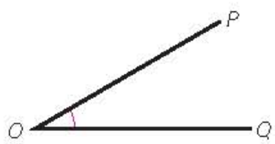
(b) **(i)** Using the result of **(a)**, join AD. Find $\angle BAD + \angle ADC$ with a protractor.

(ii) Find $\angle ABC + \angle BCD + \angle BAD + \angle ADC$.

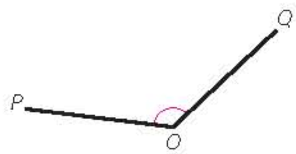
Exercise 3B

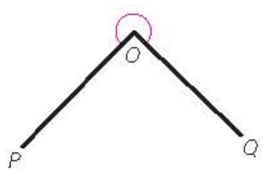
Level 1

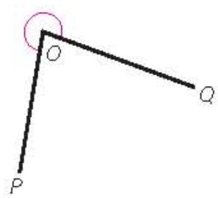
1. Find the sizes of the following marked angles with a protractor.

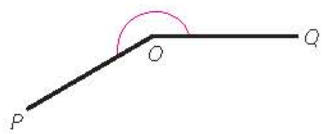
(a)


(b)


(c)


(d)


(e)


(f)


2. Draw the following angles with a protractor.

(a) 20° **(b)** 55° **(c)** 95°

(d) 120° **(e)** 210° **(f)** 300°

Level 2

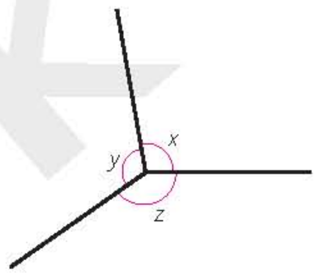
3. **(a)** Draw a right angle AOB with a protractor.

(b) Mark D within $\angle AOB$, then join OD and measure $\angle AOD$.

(c) Using the result of **(b)**, find $\angle BOD$.

4. **(a)** Find x, y and z in the figure with a protractor.

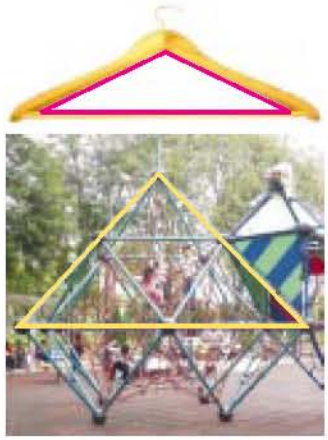
(b) Using the result of **(a)**, find $x + y + z$. Is your answer reasonable? Explain briefly.



3.4 Triangles

In geometry, a plane has no thickness. It can be extended infinitely.

> A **triangle** is a plane figure enclosed by three non-parallel line segments on the same plane.



Triangles are commonly seen in daily life.

As shown in Figure 3.12, triangle ABC has 3 vertices (A, B and C), 3 sides (AB, BC and CA) and 3 **interior angles** ($\angle ABC$, $\angle BCA$ and $\angle CAB$).

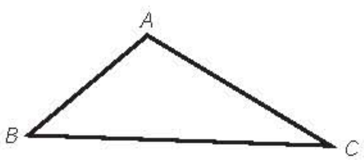


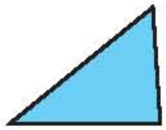
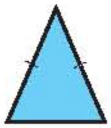
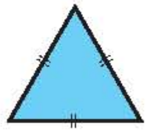
Figure 3.12

Note: 'Triangle ABC' is often denoted by 'ΔABC'.

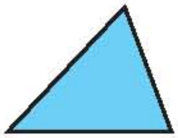
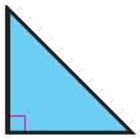
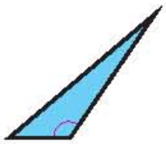
triangle 三角形 interior angle 內角

A Types of triangles

Triangles can be classified into three types according to the lengths of their sides as follows.

Type	Scalene triangle	Isosceles triangle	Equilateral triangle
Diagram			
Property	The lengths of all sides are different	The lengths of two sides are equal	The lengths of all sides are equal

Triangles can also be classified into three types according to the sizes of their interior angles.

Type	Acute-angled triangle	Right-angled triangle	Obtuse-angled triangle
Diagram			
Property	All interior angles are acute angles	One interior angle is a right angle	One interior angle is an obtuse angle

Note: Use the same mark to indicate sides of the same length.
e.g. In Figure 3.13, $AF = CD$, $AB = ED$ and $BC = FE$.

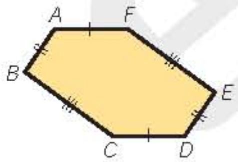


Figure 3.13

Extension 3.6 Solution

Draw an isosceles obtuse-angled triangle.

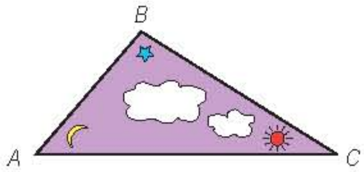
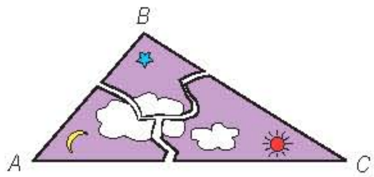
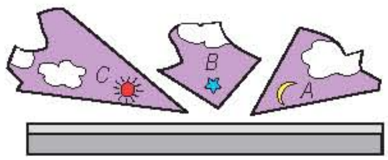
scalene triangle 不規則三角形
acute-angled triangle 銳角三角形
isosceles triangle 等腰三角形
right-angled triangle 直角三角形
equilateral triangle 等邊三角形
obtuse-angled triangle 鈍角三角形

B Angle sum of a triangle

Class Activity 3.1

Aim: To explore the property of the angle sum of a triangle

Tool required: A piece of colour paper, colour pens, a ruler, a pair of scissors or a cutter

Step 1	Draw a triangle with different patterns at its corners on a piece of colour paper. Here we use a star, a moon and a sun to identify the 3 interior angles.	
Step 2	Cut the triangle into 3 pieces such that each piece contains only one of the vertices of the triangle.	
Step 3	Put the three corners together against the side of a ruler.	

1. Do all angles line up to form a straight angle? Show Ans.

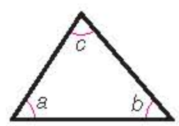
☐ Yes ☐ No

2. The sum of the three interior angles of a triangle = ______________. Show Ans.

I.T. Tool

From the above Class Activity, we can find that:

> The sum of all the interior angles of a triangle is 180°.

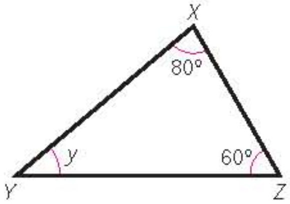


$a + b + c = 180°$

Additional Example 3.3

Example 3.3 Finding an unknown interior angle of a triangle

In the figure, find y.



Solution

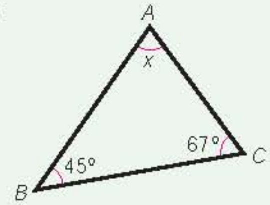
In ΔXYZ,

$$\begin{aligned} y + 80° + 60° &= 180° \\ y + 140° &= 180° \\ y &= 180° - 140° \\ &= \underline{\underline{40°}} \end{aligned}$$

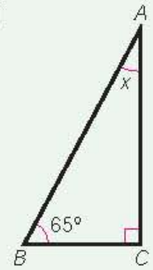
Classwork 3.3 Solution

Find x in each of the following figures.

(a)



(b)



3.5 Polygons

A **polygon** is a plane figure enclosed by three or more line segments. The name of a polygon is determined by the number of sides it has. For a polygon with n sides, we call it an ***n*-gon**.

Can you imagine a world without polygons?

Figure 3.14 shows some examples of polygons and their names.

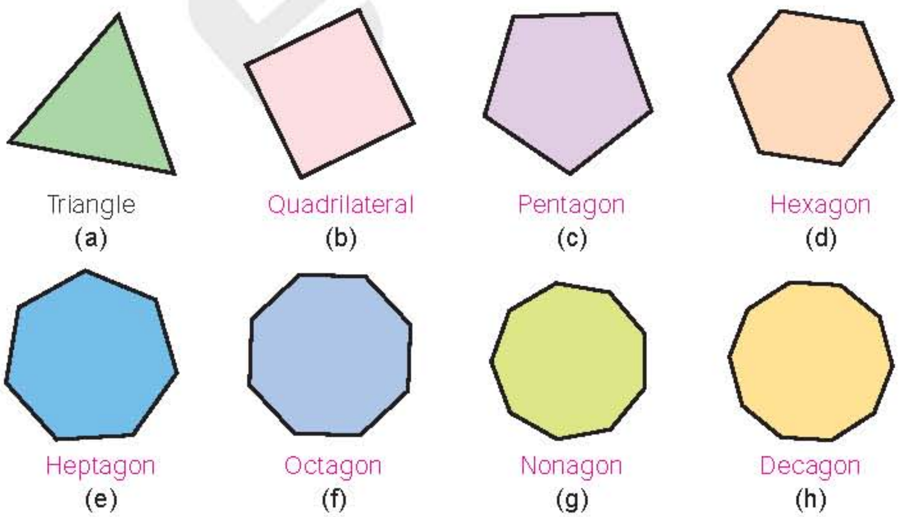


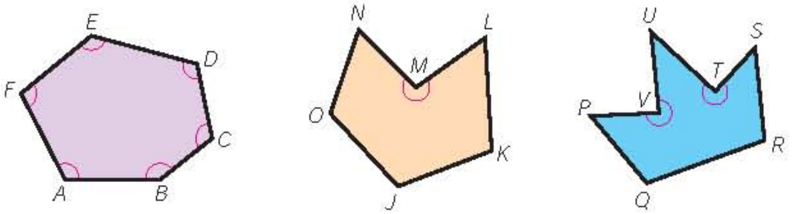
Figure 3.14

polygon 多邊形
pentagon 五邊形
octagon 八邊形
n-gon n 邊形
hexagon 六邊形
nonagon 九邊形
quadrilateral 四邊形
heptagon 七邊形
decagon 十邊形

The following are some terms describing polygons.

1. A **convex polygon** is a polygon whose interior angles are all less than 180°. (See Figure 3.15(a))

2. A **concave polygon** is a polygon with at least one interior angle greater than 180°. (See Figure 3.15(b) and (c))



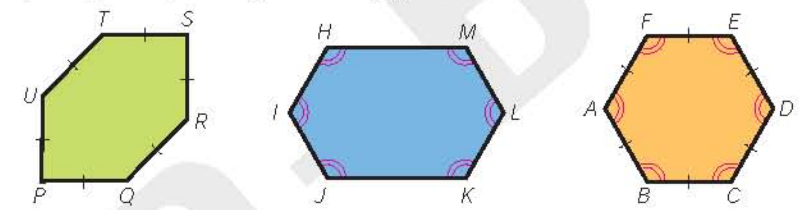
Convex hexagon (a) | Concave hexagon (b) | Concave heptagon (c)

Figure 3.15

3. An **equilateral polygon** is a polygon whose sides are all of the same length. (See Figure 3.16(a))

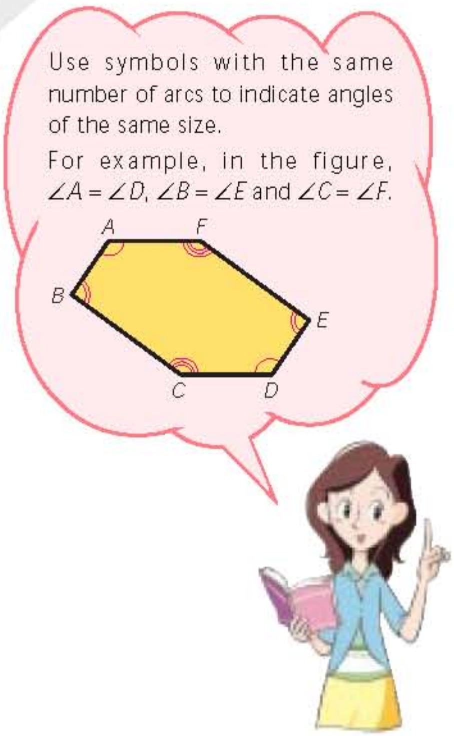
4. An **equiangular polygon** is a polygon whose interior angles are all of the same size. (See Figure 3.16(b))

5. A **regular polygon** is a polygon which is both equilateral and equiangular. (See Figure 3.16(c))



Equilateral hexagon (a) | Equiangular hexagon (b) | Regular hexagon (c)

Figure 3.16



6. A **diagonal** of a polygon is a line segment joining two vertices which are not next to each other.

   e.g. In Figure 3.17, AC and BD are diagonals of quadrilateral $ABCD$.
   AC and BD intersect at E.

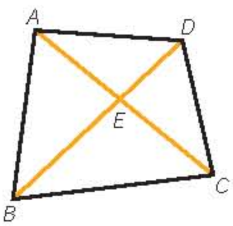


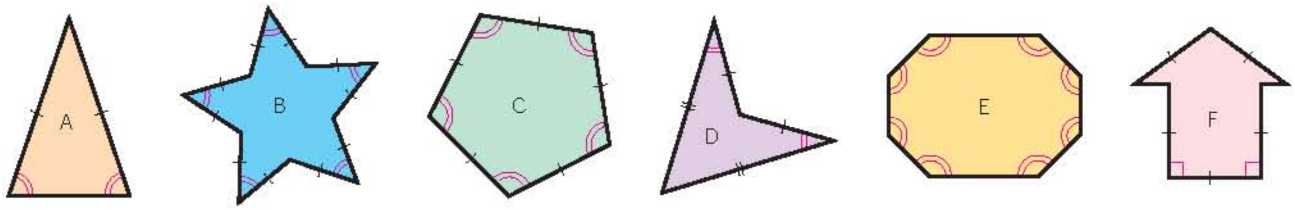
Figure 3.17

convex polygon 凸多邊形
concave polygon 凹多邊形
equilateral polygon 等邊多邊形
equiangular polygon 等角多邊形
regular polygon 正多邊形
diagonal 對角綫

Skills Upgrading Corner 3.3

Solution

1. Consider the following figures and complete the table.



		Type				
Polygon	Number of sides	Convex polygon	Concave polygon	Equilateral polygon	Equiangular polygon	Regular polygon
A	3	✓				
B						
C						
D						
E						
F						

Solution

2. **(a)** Is it true that equilateral polygons must be regular polygons? If not, give an example.

(b) Is it true that equiangular polygons must be regular polygons? If not, give an example.

Exercise 3C

Level 1

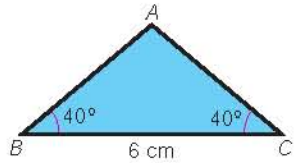
1. For each of the following conditions, draw a triangle and classify it according to the size of its interior angles.

(a) With a right angle (Mark this angle clearly)

(b) With an interior angle of 150° (Mark this angle clearly)

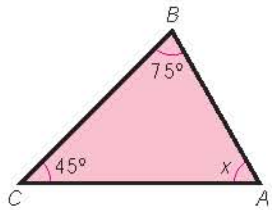
2. **(a)** According to the figure, draw ΔABC in actual size and measure the lengths of AB and AC.

(b) According to the lengths of sides of ΔABC, which type of triangle is it?

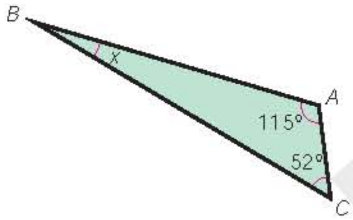


3. In each of the following figures, find x.

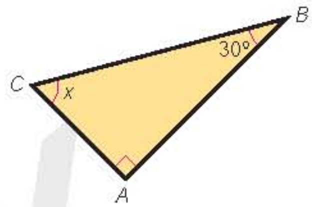
(a)



(b)



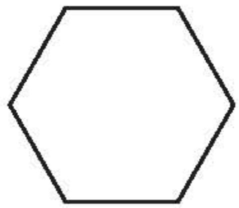
(c)



4. Find the remaining interior angle of ΔABC if two of the interior angles are given as follows:

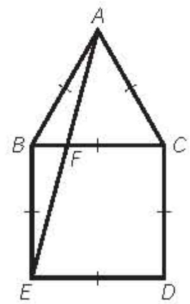
(a) $\angle A = 28°$, $\angle B = 37°$ **(b)** $\angle A = 56°$, $\angle C = 48°$ **(c)** $\angle B = 68°$, $\angle C = 68°$

5. Draw all diagonals of the hexagon in the figure. How many diagonals are there?
[A copy of the figure is provided in the Appendix.]



6. In the figure, $BCDE$ is a square. It is known that $AB = BC = CA$, and AE and BC intersect at F. Write down the triangle(s) under each of the following types.

(a) Acute-angled triangle
(b) Obtuse-angled triangle
(c) Right-angled triangle
(d) Scalene triangle
(e) Isosceles triangle
(f) Equilateral triangle



Level 2

7. Draw a hexagon for each of the following conditions, and mark all the acute angles.

(a) One interior angle is acute, while the others are obtuse.
(b) Two interior angles are acute, while the others are obtuse.

8. **(a)** Can a triangle have one right angle and one obtuse angle? Explain briefly.

(b) Can all interior angles of a triangle be smaller than 60°? Explain briefly.

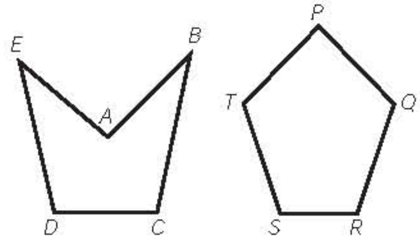
9. **(a)** In the two pentagons on the right, draw all the diagonals with points A and P as vertices.

(b) For each figure in **(a)**, how many diagonals are drawn? How many triangles are formed?

(c) Using the result of **(b)**, find the sum of all interior angles of a pentagon.

(d) Can a pentagon have two reflex angles as interior angles? If yes, give an example and mark the reflex angles.

[A copy of each figure is provided in the Appendix.]



3.6 Solids

A Basic concepts of solids

> All **solids** occupy space and are bounded by one or more surfaces.

The surfaces bounding a solid can be planes or curved surfaces.

The following shows some common solids in daily life.

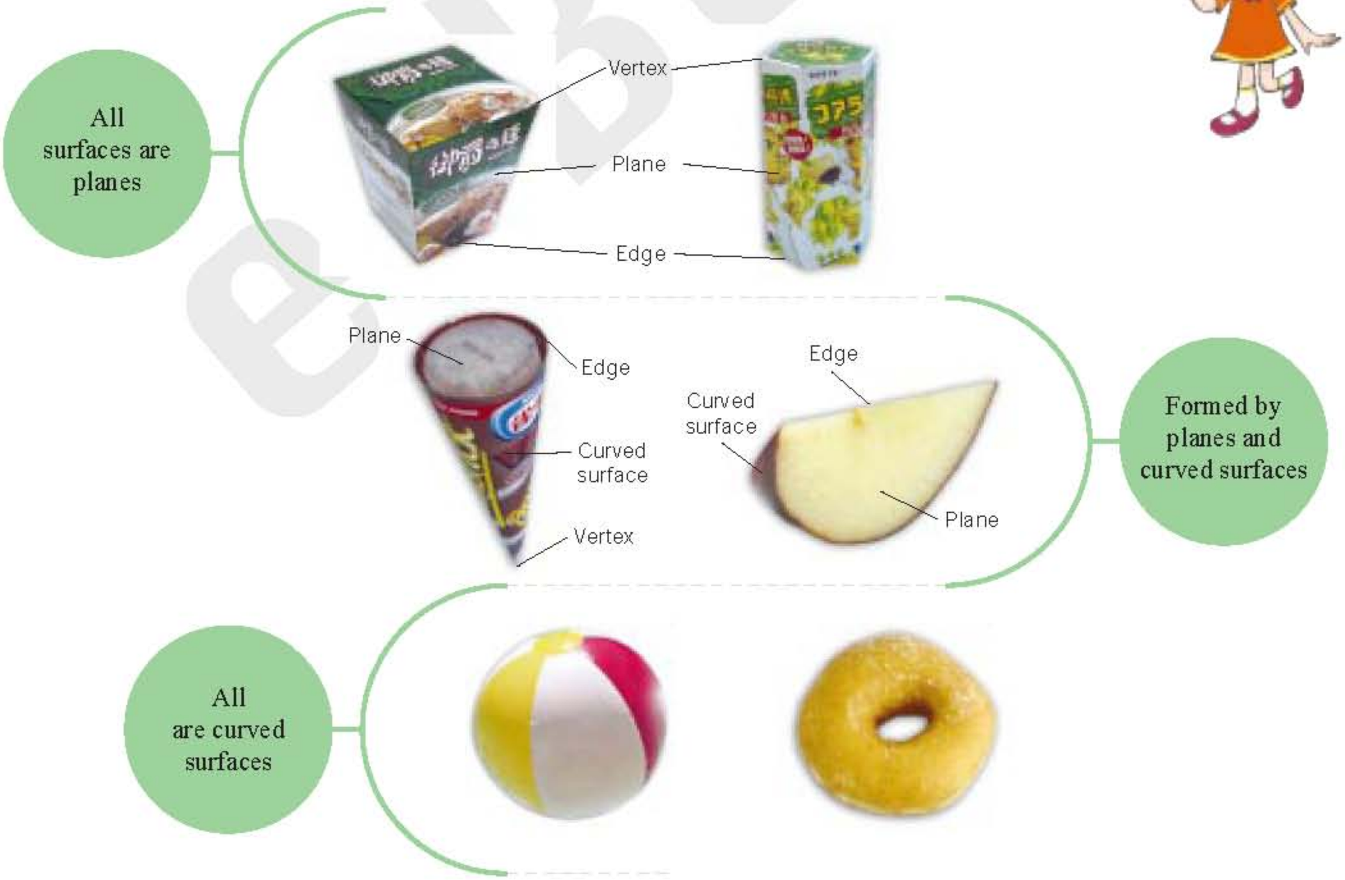


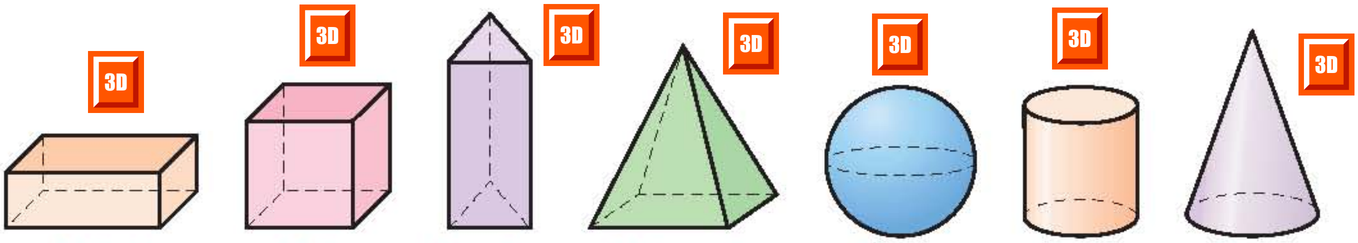
Figure 3.18

solid 立體

Extension 3.7

Solution

Consider the structures of the following solids, and complete the table.



Structure	All surfaces are planes	Formed by planes and curved surfaces	All are curved surfaces
Solid			

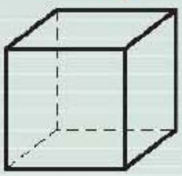
B Cross-sections of a solid

Class Activity 3.2

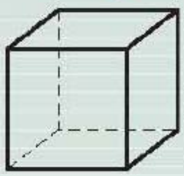
Aim: To explore the **cross-sections** of a solid

The photo shows a cubic *bean curd*. If it is cut only once, draw the cross-section in the shape as indicated in each of the following figures.

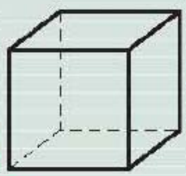
(a) Isosceles triangle

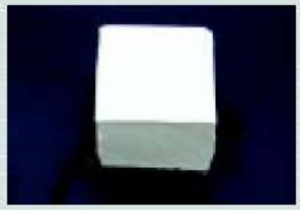


(b) Rectangle

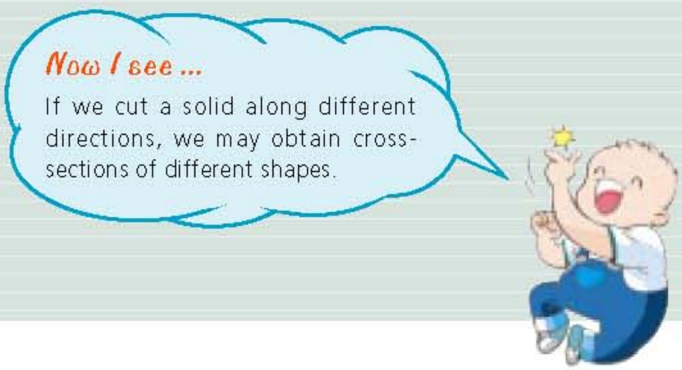


(c) Trapezium





Show Ans.



I.T. Tool

cuboid 長方體
rectangular pyramid 長方錐體
cone 圓錐體
cube 正方體
sphere 球體
cross-section 截面
triangular prism 三角柱體
cylinder 圓柱體

bean curd 豆腐

For some solids, if any of them is cut along one direction, cross-sections of the same shape and size at different positions will be obtained. (See Figure 3.19)

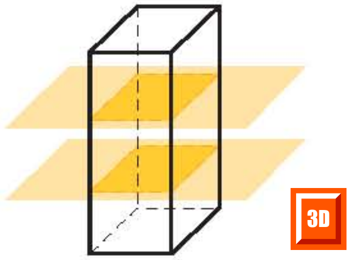


Figure 3.19

> If we cut a solid along one direction, the cross-sections obtained at any positions are the same in both shape and size, these cross-sections are called **uniform cross-sections**.

e.g. The following solids have uniform cross-sections. (See Figure 3.20)

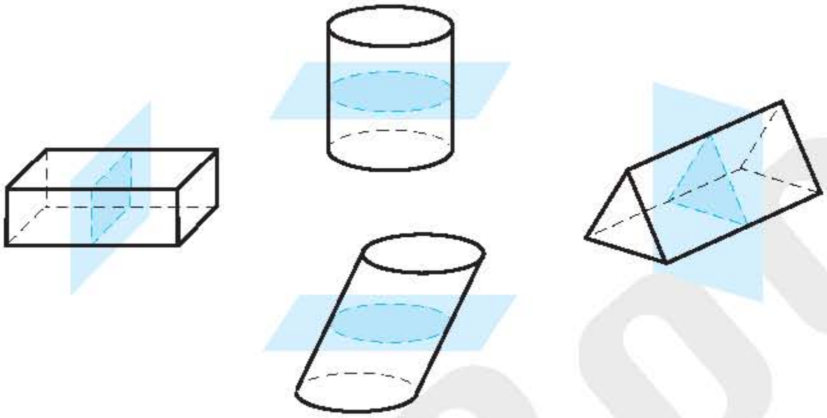
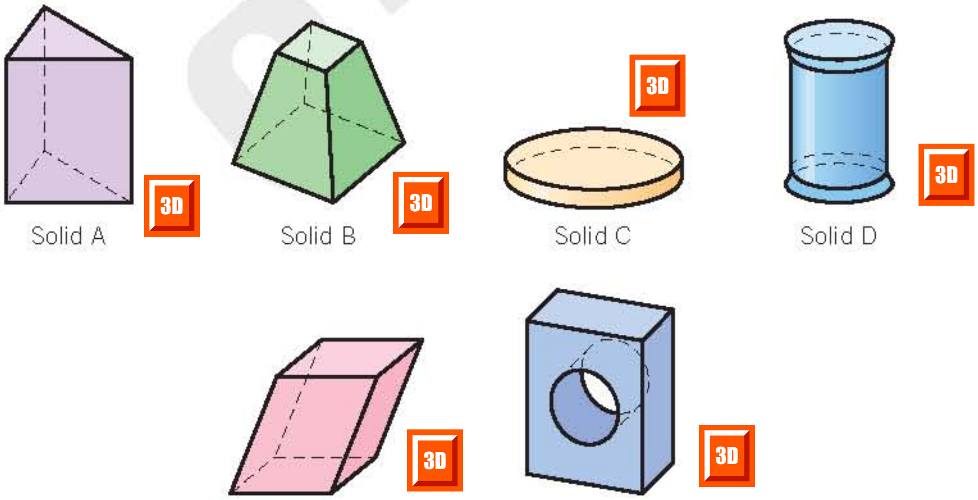


Figure 3.20

Extension 3.8

Solution

In the following figures, which solids have uniform cross-sections?



Solid A Solid B Solid C Solid D

Solid E Solid F

uniform cross-section 均匀截面

C Polyhedra

Polyhedra are closed solids which are formed by polygons.

In a polyhedron, each face is a polygon, each edge is a line segment. The following are some examples of polyhedra.

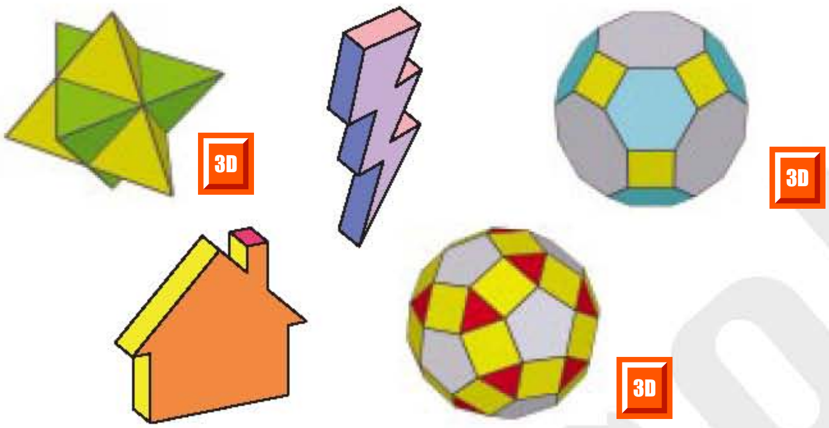


Figure 3.21

D Regular polyhedra

If all faces of a polyhedron are the same regular polygon and each vertex is an intersection of the same number of edges, it is called a **regular polyhedron**.

In the 3rd century BC, Plato (427BC - 347BC), a great Greek philosopher, and his students studied the following 5 kinds of convex regular polyhedra called **Platonic solids**.

Table 3.5 shows the name and shape of each regular polyhedron.

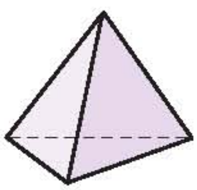
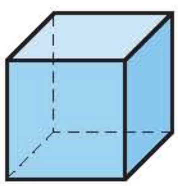
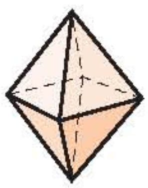
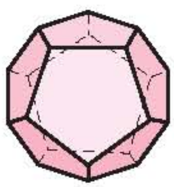
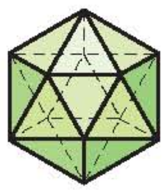
Name	Regular tetrahedron	Regular hexahedron (Cube)	Regular octahedron	Regular dodecahedron	Regular icosahedron
Shape					

Table 3.5

I.T. Tool

Note: The nets of the polyhedra above are provided in the Appendix. Use the nets to make different regular polyhedra.

polyhedron 多面體
regular tetrahedron 正四面體
regular dodecahedron 正十二面體
regular polyhedron 正多面體
regular hexahedron 正六面體
regular icosahedron 正二十面體
Platonic solid 柏拉圖立體
regular octahedron 正八面體

E Euler's formula

Class Activity 3.3

Aim: To investigate the relation among the number of vertices, edges and faces of a polyhedron

1. Consider Figure I to VI, write down the number of vertices, edges and faces of each polyhedron in the following table.

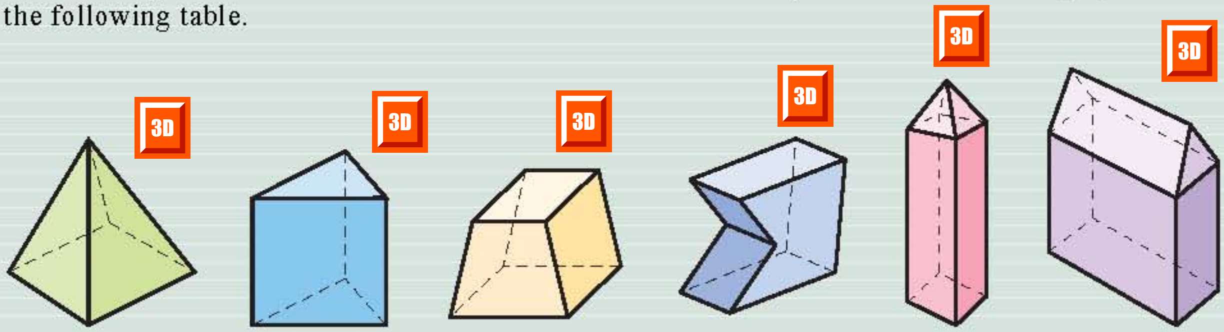


Figure I Figure II Figure III Figure IV Figure V Figure VI

Show Ans.

	Figure I	Figure II	Figure III	Figure IV	Figure V	Figure VI
Number of vertices V						
Number of edges E						
Number of faces F						
$V - E + F$						

2. What is the pattern of the values of $V - E + F$? Show Ans.

____

The polyhedra in Class Activity 3.3 satisfy the following property:

If the number of vertices, number of edges and number of faces of a polyhedron are V, E and F respectively, then

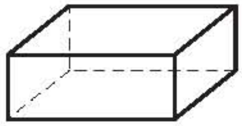
$$\boxed{V - E + F = 2}$$

This formula is called **Euler's formula**.

Euler's formula 歐拉公式

Skills Upgrading Corner 3.4

Solution **1.** The figure shows a cuboid. How many different uniform cross-sections does it have? Copy the figure and shade the different uniform cross-sections.



Solution **2.** Referring to Table 3.5 on page 3.24, check whether the regular octahedron satisfies Euler's formula.

Exercise 3D

Level 1

1. Write down the shapes of the following objects.

(a)

(b)

(c)

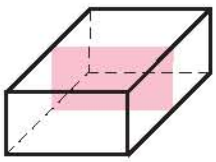
(d)

(e)

(f)

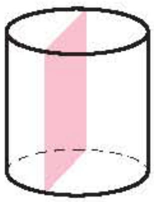
2. The following shaded regions are cross-sections of the solids. Write down the shapes of the cross-sections.

(a)



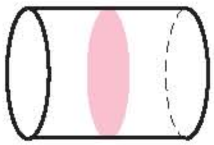
Cuboid

(b)



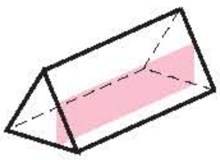
Cylinder

(c)



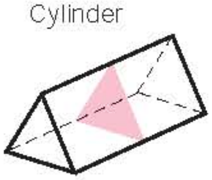
Cylinder

(d)



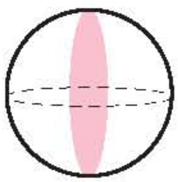
Triangular prism

(e)



Triangular prism

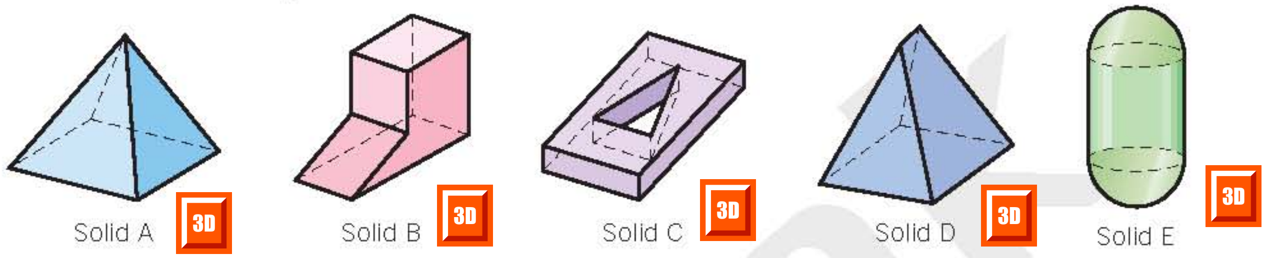
(f)



Sphere

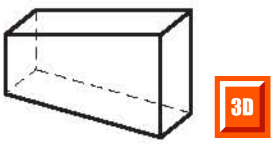
3. Which of the following solids are polyhedra?

4. Which of the following solids have uniform cross-sections?

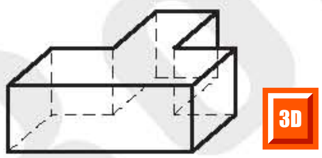


5. Colour the uniform cross-section of each of the following solids.
[A copy of each figure is provided in the Appendix.]

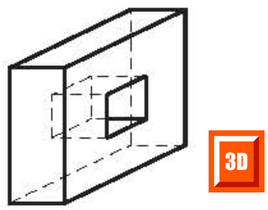
(a)



(b)



(c)



Level 2

6. Figures I and II show two solids formed by cutting off 1 corner and 2 corners respectively from a cube.

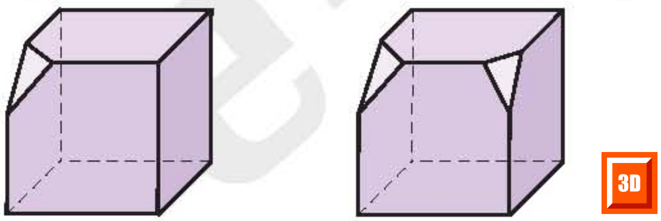


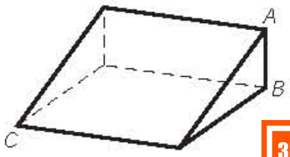
Figure I　　Figure II

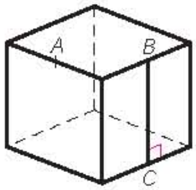
(a) Complete the following table.

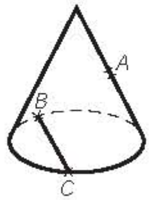
Type of solid	Number of vertices V	Number of edges E	Number of faces F	$V - E + F$
Cube with 1 corner cut off				
Cube with 2 corners cut off				
Cube with 3 corners cut off				

(b) Do the solids above satisfy Euler's formula?

7. In each of the following, a cross-section is obtained by cutting through the points A, B and C. Colour the cross-section of each figure.
[A copy of each figure is provided in the Appendix.]

(a)


(b)


(c)


3.7 2-Dimensional Representations of Solids

A Outlines of solids

Class Activity 3.4

Aim: To explore 3-dimensional objects

The following are photos of an object taken from different directions and their corresponding outlines.


Figure I

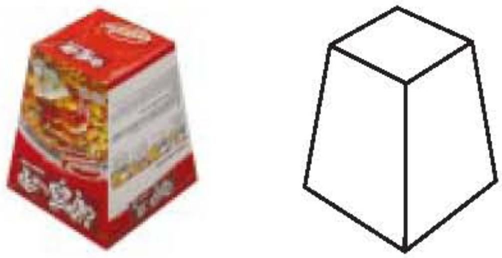


Figure II


Figure III

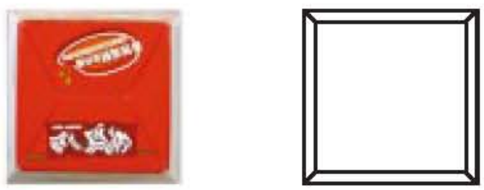


Figure IV

1. Figure ________ gives you the strongest sense of 3-dimension.

2. Collect the results in question **1** from all your classmates, and record them in the following table.

Figure	I	II	III	IV
Number of students				

3. Among the whole class, Figure ________ is considered to have the strongest sense of 3-dimension.

Now I see ...

Photos or outlines showing clearly the length, width and height of an object can give a stronger sense of 3-dimension.

Here are two types of grids which are commonly used in drawing plane figures of solids.

B Isometric grids

> An **isometric grid** is a tool for drawing the 2-dimensional representations of solids. Its grid lines form equilateral triangles.

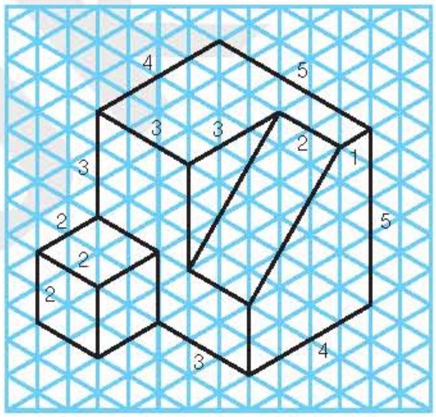


Figure 3.22

Figure 3.23 shows a cube with sides of 2 units each. Table 3.6 shows the steps of drawing the cube on an isometric grid.
(Assume the length of each line segment on an isometric grid is 1 unit.)

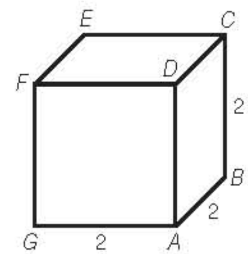
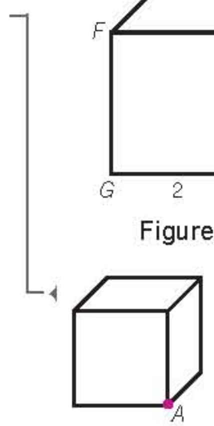


Figure 3.23



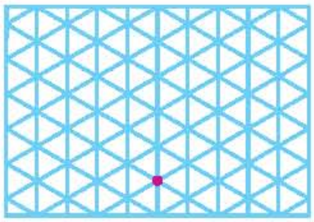
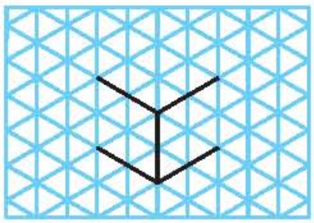
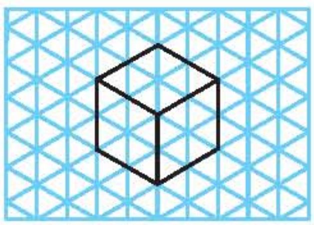
Step 1	Choose a vertex from the base of the cube to be the lowest point on the drawing. Here we choose point A. Mark it (i.e. point A) on the grid.	
Step 2	Draw a vertical line of 2 units from the lowest point. Draw 2 sets of parallel lines of 2 units at each end of the vertical line.	
Step 3	Draw the rest of the line segments by following the grid lines, and complete the drawing of the cube.	

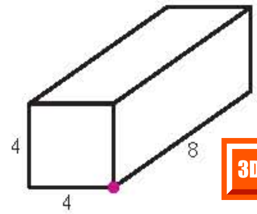
Table 3.6

isometric grid 等距方格

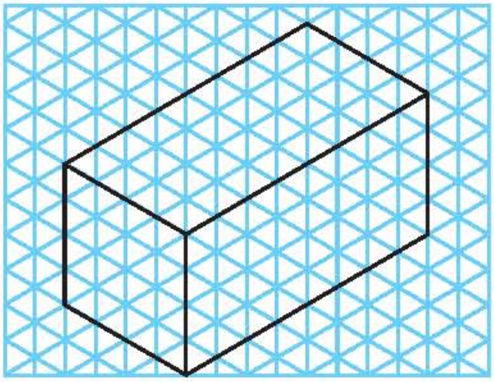
Additional Example 3.4

Example 3.4 Drawing a solid on an isometric grid

Treat the purple spot as the lowest point of the given solid, draw the solid on an isometric grid. (The numbers in the figure are lengths of sides.)



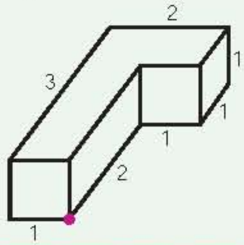
Solution

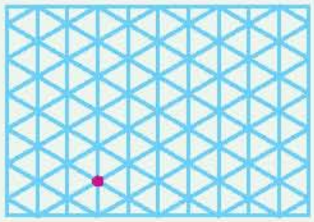


Classwork 3.4

Solution

Treat the purple spot as the lowest point of the given solid, draw the solid on an isometric grid. (The numbers in the figure are lengths of sides.)





C Oblique grids

An **oblique grid** is another tool for drawing 2-dimensional representations of solids. Its grid lines form isosceles right-angled triangles.

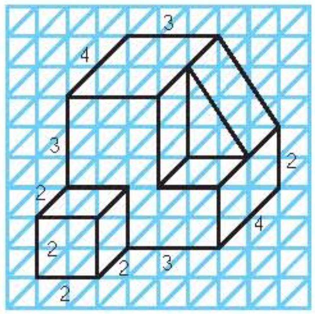


Figure 3.24

Figure 3.25 shows a cube with sides of 2 units each. Table 3.7 shows the steps of drawing the cube on an oblique grid.
(Assume the lengths of all horizontal and vertical line segments on an oblique grid are 1 unit.)

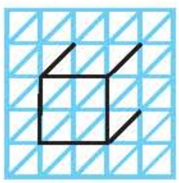
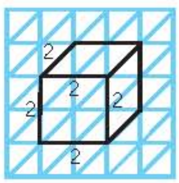
Step 1	Draw a square with sides of 2 units each on the oblique grid. This is the front surface of the cube. Draw the sides of lateral faces by following the inclined lines.	
Step 2	Complete the drawing by joining the inclined lines with vertical and horizontal lines.	

Table 3.7

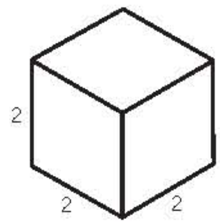


Figure 3.25

Each inclined line segment is fixed at 2 units to make the figure looks more reasonable.

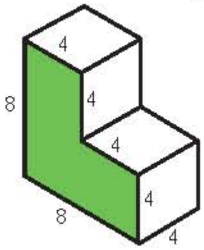
I.T. Tool

oblique grid 斜網格

Additional Example 3.5

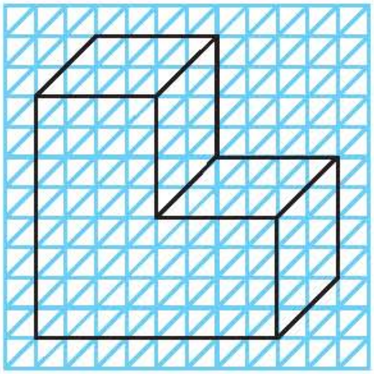
Example 3.5 Drawing a solid on an oblique grid

Treat the green surface as the front surface of the given solid, draw the solid on an oblique grid. (The numbers in the figure are lengths of sides.)



3D

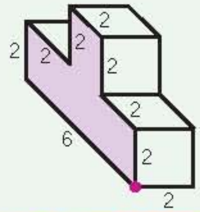
Solution



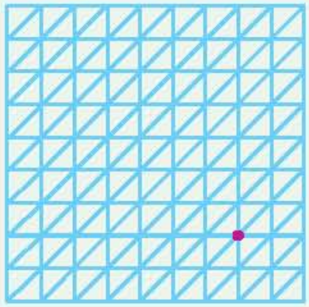
Classwork 3.5

Solution

Treat the purple surface as the front surface of the given solid, draw the solid on an oblique grid. (The numbers in the figure are lengths of sides.)



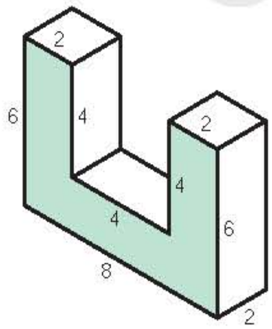
3D



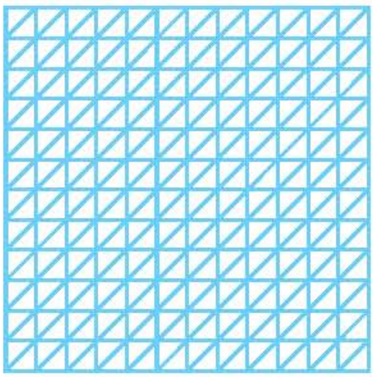
Skills Upgrading Corner 3.5

Solution

1. Treat the green surface as the front surface of the given solid, draw the solid on an oblique grid. (The numbers in the figure are lengths of sides.)

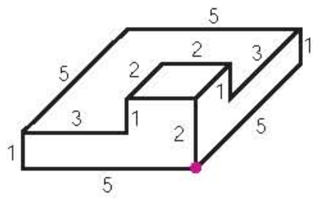


3D

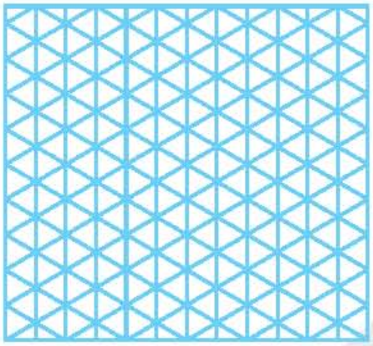


Solution

2. Treat the purple spot as the lowest point of the given solid, draw the solid on an isometric grid. (The numbers in the figure are lengths of sides.)



3D



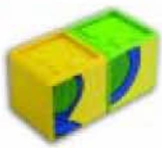
Exercise 3E

[Isometric grids and oblique grids are provided in the Appendix.]

Level 1

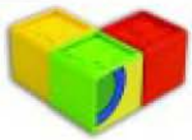
1. The following solids are formed by cubes with sides of 1 unit each. Draw them on isometric grids.

(a)



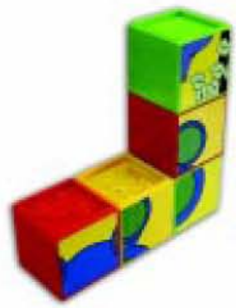
3D

(b)



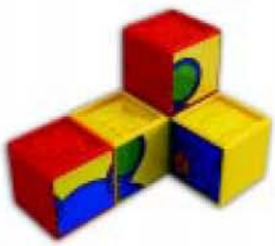
3D

(c)



3D

(d)

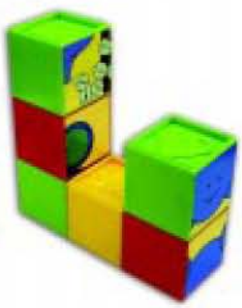


3D

(e)

3D

(f)



3D

2. The following solids are formed by cubes with sides of 2 units each. Draw them on oblique grids.

(a)


(b)


(c)

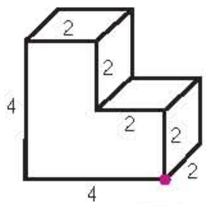

(d)

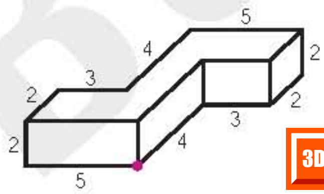

(e)

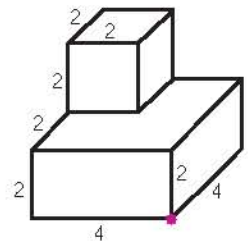

(f)


Level 2

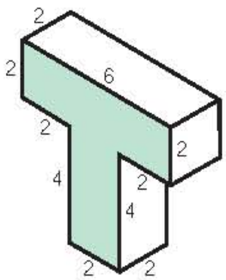
3. Treat the purple spot as the lowest point of each of the following solids, draw the solids on isometric grids. (The numbers in the figures are lengths of sides.)

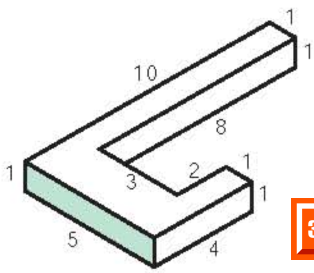
(a)


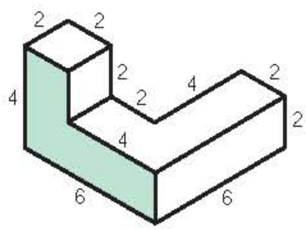
(b)


(c)


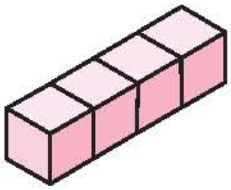
4. Treat the green surface as the front surface of each of the following solids, draw the solids on oblique grids. (The numbers in the figures are lengths of sides.)

(a)


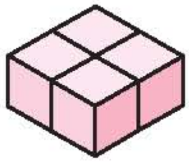
(b)


(c)


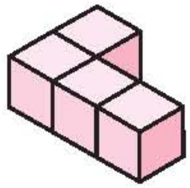
5. **(a)** Try to form a new solid by combining solids B and C, then draw it on an isometric grid.



Solid A



Solid B



Solid C

(b) It is known that when 4 cubes with sides of 2 units each are put together with their connecting faces completely overlap, 8 different solids can be obtained. Three of them are shown in **(a)**. Draw the other 5 solids on oblique grids.

3.8 Drawing Tools

Figure 3.26 shows some drawing tools: a ruler, a pair of **compasses** and **set squares**.

Figure 3.26

A Rulers and compasses

A ruler is a tool for drawing and measuring the lengths of line segments. A pair of compasses is a tool for drawing circles. In primary school, we have learned how to draw circles using a pair of compasses.

compasses 圓規   set square 三角尺

Extension 3.9

Solution

Draw two circles using P and Q as centres and the length of AB as the radius.

B Set squares

There are two common types of set squares. The interior angles of one of them are 45°, 45° and 90°, and the other one are 30°, 60° and 90°. (See Figure 3.27)

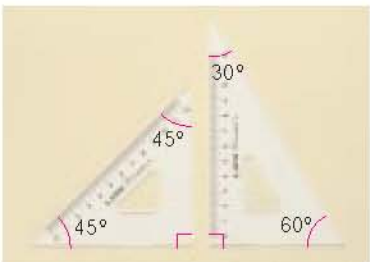


Figure 3.27

We can draw **parallel lines** and **perpendicular lines** with a ruler and a set square.

I. Drawing parallel lines

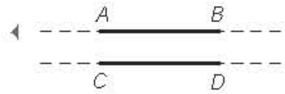
> If two straight lines on a plane never intersect no matter how far they are extended, these two lines are parallel lines.

Rails are daily examples of parallel lines.

parallel lines 平行綫　　perpendicular lines 垂直綫

In Figure 3.28, AB and CD are parallel lines.

Figure 3.28



AB and CD will not intersect even when they are extended.

We can use the symbol '//' to represent 'parallel to', i.e. 'AB // CD' means 'AB is parallel to CD'.

Table 3.8 shows the steps of drawing a straight line parallel to a given line AB.

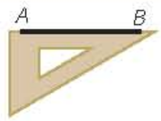
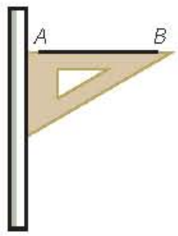
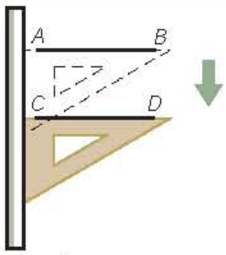
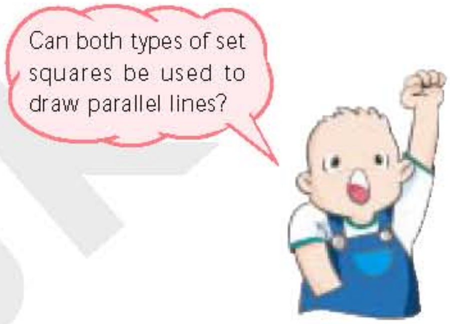
Step 1	Put one side of a set square against AB.	
Step 2	Put a ruler along the other side of the set square.	
Step 3	Fix the ruler. Slide the set square along the ruler. Then draw a straight line CD along the set square.	 (Ruler fixed)
Step 4	A straight line CD which is parallel to AB is drawn.	

Table 3.8



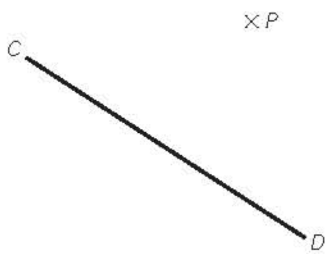
Mark the two lines with the same number of arrowheads in the same direction to indicate that they are parallel.

I.T. Tool

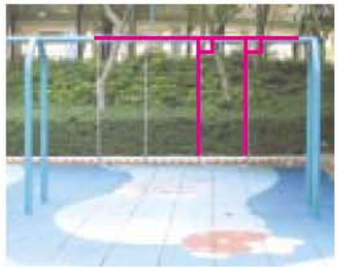
Extension 3.10

Solution

Draw a straight line which passes through P and is parallel to CD.



II. Drawing perpendicular lines



> If two straight lines on a plane intersect at an angle of 90°, they are perpendicular to each other.

In Figure 3.29(a), 'AB is perpendicular to CD'. We use the symbol '⊥' to represent 'perpendicular to', i.e. $AB \perp CD$.

Generally, we use a right angle to indicate lines that are perpendicular to each other. (See Figure 3.29)

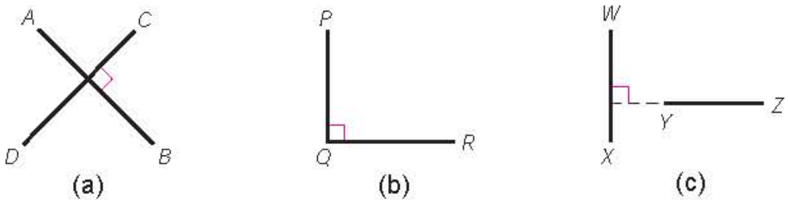


Figure 3.29

Table 3.9 shows the steps of drawing a straight line which passes through a point R and is perpendicular to AB.

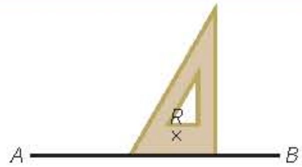
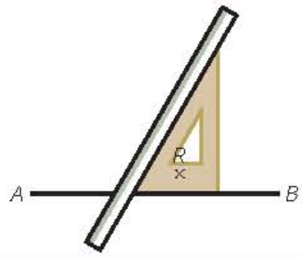
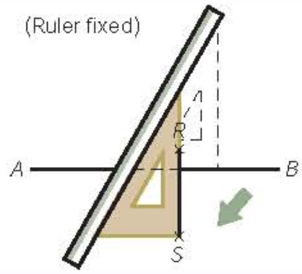
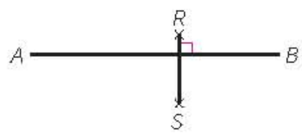
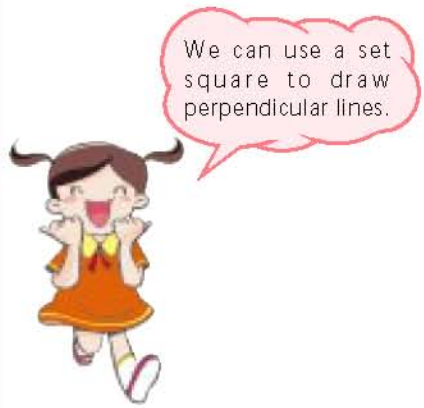
Step 1	Put a side of a set square with a right angle against AB.	
Step 2	Put a ruler along the longest side of the set square.	
Step 3	Fix the ruler. Slide the set square along the ruler until its side reaches point R. Draw a straight line RS along the set square.	
Step 4	A straight line RS which is perpendicular to AB is drawn.	

Table 3.9



Note: In Figure 3.30, Y is a point on AB such that $XY \perp AB$. We call point Y the **foot of perpendicular** from X to AB.

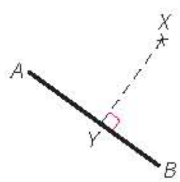


Figure 3.30

I.T. Tool

foot of perpendicular 垂足

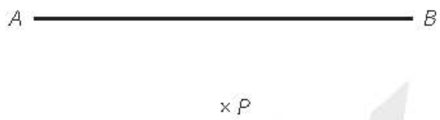
Extension 3.11

Solution

In each of the following, draw a straight line which passes through point P and is perpendicular to AB.

(a)

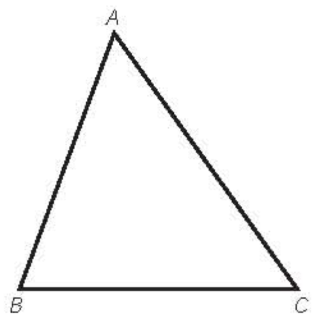
(b)



Skills Upgrading Corner 3.6

Solution

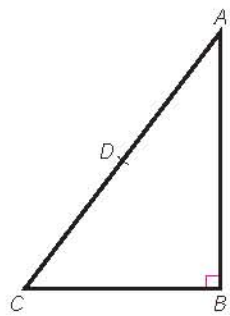
1. The figure shows ΔABC.



   (a) Draw a straight line passing through A and perpendicular to BC.
   (b) Draw a straight line passing through B and perpendicular to AC.
   (c) Draw a straight line passing through C and perpendicular to AB.
   (d) Do the lines drawn above intersect at one point?

Solution

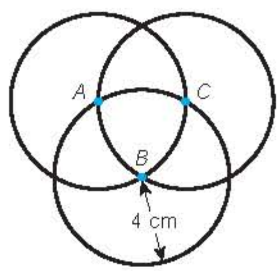
2. In the figure, ABC is a right-angled triangle. D is a point on AC.



   (a) Draw a straight line which passes through D and is parallel to BC. Mark the point of intersection of this line and AB as P.
   (b) Draw a straight line which passes through D and is parallel to AB. Mark the point of intersection of this line and BC as Q.
   (c) What is the shape of quadrilateral $PDQB$?

Solution

3. In the figure, A, B and C are centres of three circles with radii of 4 cm each. Draw this figure in actual size.

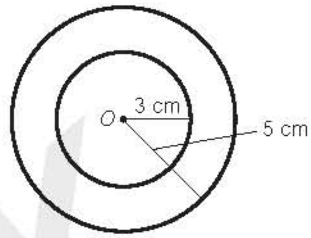


Exercise 3F

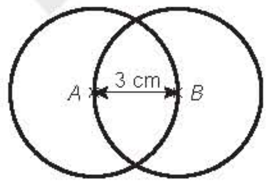
[In this exercise, only rulers, compasses and set squares can be used for drawing.]

Level 1

1. In the figure, the two circles share the same centre O. The radii of the two circles are 3 cm and 5 cm. Draw the figure in actual size.



2. In the figure, the radii of two circles are 3 cm. Their centres A and B are 3 cm apart. Draw the figure in actual size.

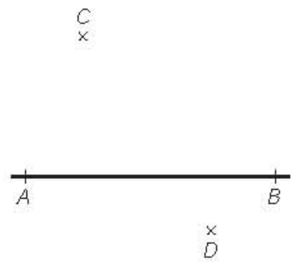


3. **(a)** In the figure, draw a straight line which passes through C and is parallel to AB.

(b) In the figure, draw a straight line which passes through D and is parallel to AB.

(c) What is the relation between the lines drawn in **(a)** and **(b)**?

[A copy of the figure is provided in the Appendix.]

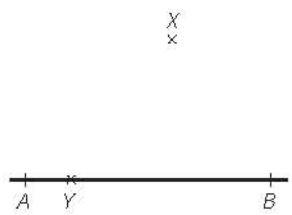


4. **(a)** In the figure, draw a straight line which passes through X and is perpendicular to AB.

(b) In the figure, draw a straight line which passes through Y and is perpendicular to AB.

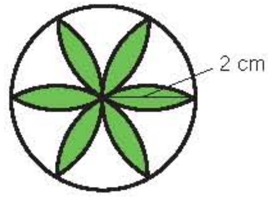
(c) What is the relation between the lines drawn in **(a)** and **(b)**?

[A copy of the figure is provided in the Appendix.]



Level 2

5. In the figure, the shaded region is formed by 6 circles with radii of 2 cm each. Draw the figure in actual size.



6. **(a)** **(i)** In the figure, draw a straight line which passes through C and is parallel to BA.

(ii) In the figure, draw a straight line which passes through A and is parallel to BC.

(b) If the two lines drawn in **(a)(i)** and **(a)(ii)** intersect at D, what is the shape of $ABCD$?

[A copy of the figure is provided in the Appendix.]

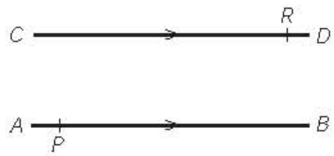
7. In the figure, AB is parallel to CD.

(a) **(i)** Draw a straight line which passes through P and is perpendicular to AB. Mark S at its point of intersection with CD.

(ii) Draw a straight line which passes through R and is perpendicular to CD. Mark Q at its point of intersection with AB.

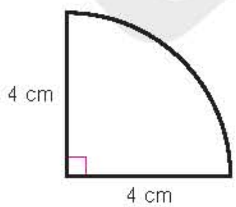
(b) What is the shape of $PQRS$?

[A copy of the figure is provided in the Appendix.]

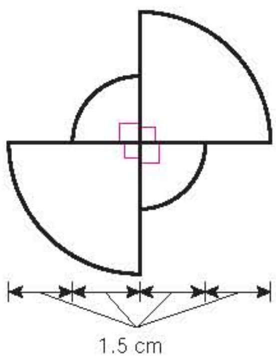


8. Draw the following figures in actual size.

(a)



(b)



Chapter Summary

Chapter Summary

A. Term Introduced

[This is a quiz to check your understanding of some special terms in this chapter. Match items in column A to column B appropriately.]

Column A			Column B
1. Scalene triangle	•	•	**(a)** A triangle with two equal sides.
2. Isosceles triangle	•	•	**(b)** A polygon with all equal interior angles.
3. Equilateral triangle	•	•	**(c)** A closed solid which is formed by polygons.
4. Convex polygon	•	•	**(d)** A polygon with all sides of equal length.
5. Concave polygon	•	•	**(e)** A triangle with three unequal sides.
6. Equilateral polygon	•	•	**(f)** A polyhedron with the same regular polygons as faces. Each vertex is an intersection of the same number of edges.
7. Equiangular polygon	•	•	**(g)** A polygon with all interior angles smaller than 180°.
8. Regular polygon	•	•	**(h)** A polygon with all equal angles and sides.
9. Polyhedron	•	•	**(i)** A triangle with all sides of equal length.
10. Regular polyhedron	•	•	**(j)** A polygon with at least one interior angle greater than 180°.

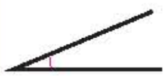
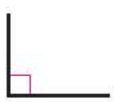
B. Fact to Remember

1. 1 round angle $= 360^\circ$

$1^\circ = 60'$

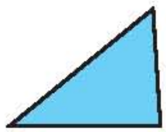
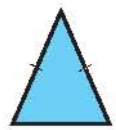
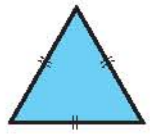
$1' = 60''$

2. Types of angles

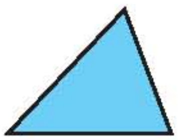
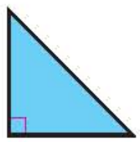
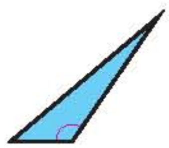
Type	Acute angle	Right angle	Obtuse angle	Straight angle	Reflex angle	Round angle
Diagram						
Size	Between 0° and 90°	90°	Between 90° and 180°	180°	Between 180° and 360°	360°

3. The sum of all the interior angles of a triangle is 180°.

4. Classification of triangles according to the lengths of their sides

Type	Scalene triangle	Isosceles triangle	Equilateral triangle
Diagram			
Property	The lengths of all sides are different	The lengths of two sides are equal	The lengths of all sides are equal

5. Classification of triangles according to the sizes of their interior angles

Type	Acute-angled triangle	Right-angled triangle	Obtuse-angled triangle
Diagram			
Property	All interior angles are acute angles	One interior angle is a right angle	One interior angle is an obtuse angle

6. Euler's formula
In a polyhedron, if the number of vertices, number of edges and number of faces are V, E and F respectively, then $V - E + F = 2$.

7. Rulers and set squares can be used to draw parallel lines and perpendicular lines.

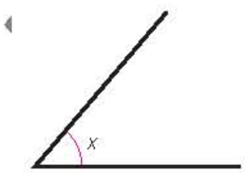
Check Yourself

[This is a quiz to remind you of the basic concepts you have learned in this chapter. Each question tests a concept under the section listed on the right. Failure in any part of a question indicates a need to do a revision on the section listed.]

Section

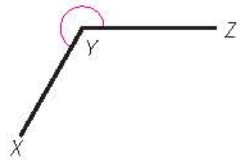
1. **(a)** In the figure, x is an acute / an obtuse / a right angle. 3.2

(b) Express $\frac{3}{4}$ of a round angle in degrees.

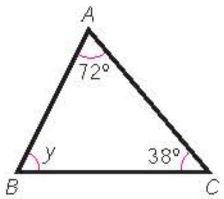


2. **(a)** Using a protractor, the size of reflex angle XYZ is ______ . 3.3

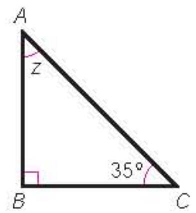
(b) Draw an angle with the size of 200°.



3. **(a)** In the figure, y = ______ . 3.4B

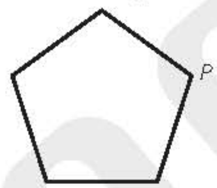


(b) In the figure, find z.

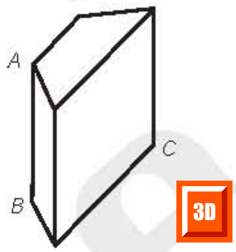


4. **(a)** The following figure shows a <u>concave / convex</u> polygon. It is <u>equilateral / equiangular</u>. 3.5

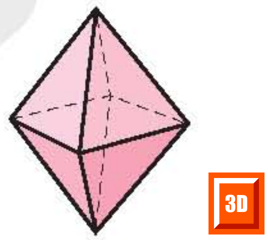
(b) In the following pentagon, draw all the diagonals from P.



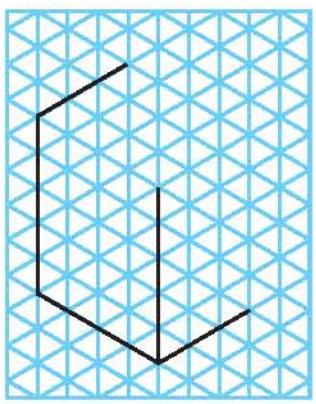
5. **(a)** The figure shows a prism. Draw the cross-section which passes through A, B and C. 3.6



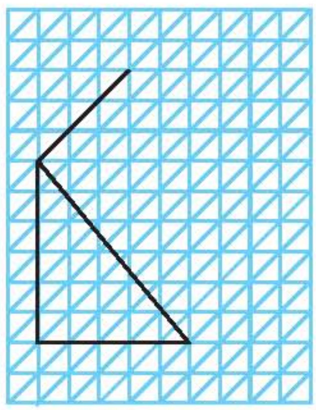
(b) In the following solid, all its faces are equilateral triangles. Write down the name of the solid.



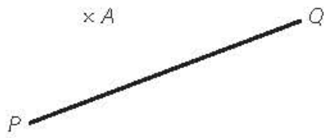
6. **(a)** Draw some straight lines on the following isometric grid to form the plane figure of a cuboid. 3.7



(b) Draw some straight lines on the following oblique grid to form the plane figure of a triangular prism.



7. 3.8



In the figure above,

(a) draw a straight line which passes through A and is parallel to PQ.

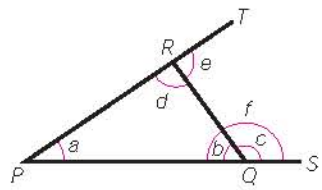
(b) draw a straight line which passes through A and is perpendicular to PQ.

Revision Exercise 3

Level 1

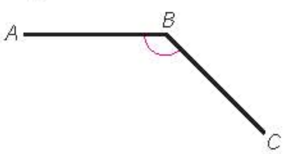
1. Referring to the figure, complete the following table.

		Name
Angle	a	$\angle RPQ$
	b	
	c	
	d	
	e	
	f	

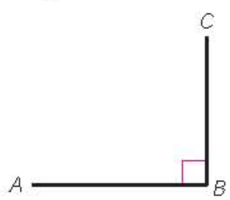


2. Classify each of the following marked angles.

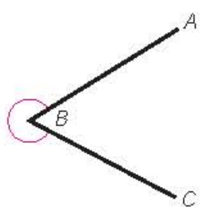
(a)



(b)

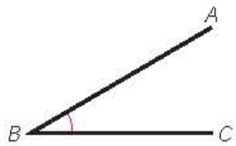


(c)

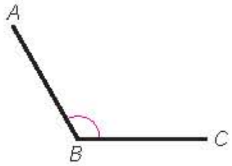


3. Find the sizes of the following angles with a protractor.

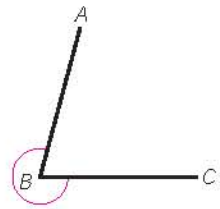
(a)



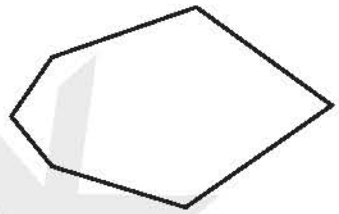
(b)



(c)



4. Draw all diagonals of the given polygon.
[A copy of the figure is provided in the Appendix.]



5. Express the following angles in degrees.

(a) 3 right angles

(b) 4 right angles

(c) $1\frac{1}{2}$ right angles

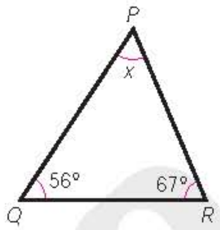
(d) $2\frac{1}{3}$ right angles

(e) $\frac{2}{9}$ of a round angle

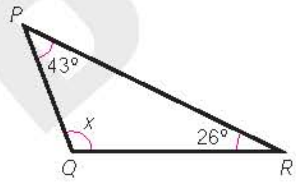
(f) $3\frac{3}{8}$ round angles

6. In each of the following figures, find x.

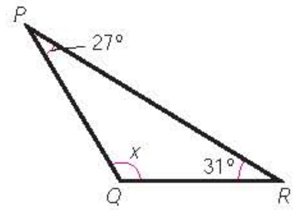
(a)



(b)



(c)



7. Find the marked angle between the minute-hand and the hour-hand of the clock as shown.

8. Two of the interior angles of a quadrilateral are 60° and 140°, and one of the sides is 3 cm. Draw two of its possible shapes.

9. It is known that each interior angle of a regular octagon is 135°. Draw a regular octagon with sides of 2 cm each.

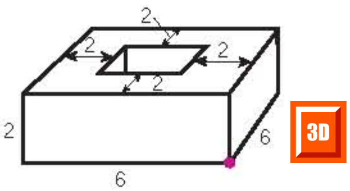
10. Which of the following are polyhedra? For each polyhedron, write down its number of vertices V, edges E and faces F.

Solid A Solid B Solid C Solid D Solid E

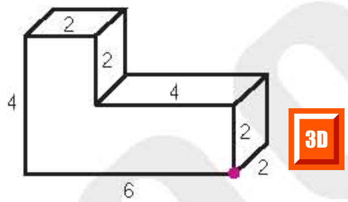
11. Treat the purple spot as the lowest point of each of the following solids, draw the solids on isometric grids (the numbers in the figures are lengths of sides). If any solid has a uniform cross-section, colour the uniform cross-section.

[Isometric grids are provided in the Appendix.]

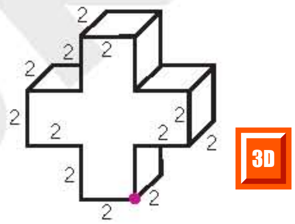
(a)



(b)



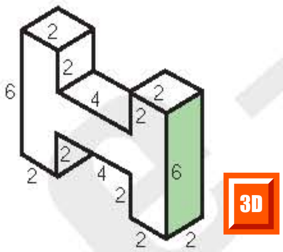
(c)



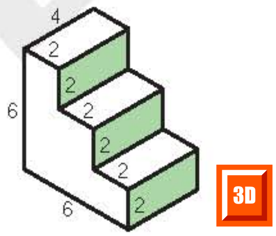
12. Treat the green surface as the front surface of each of the following solids, draw the solids on oblique grids (the numbers in the figures are lengths of sides). If any solid has a uniform cross-section, colour the uniform cross-section.

[Oblique grids are provided in the Appendix.]

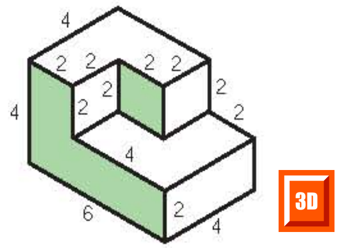
(a)



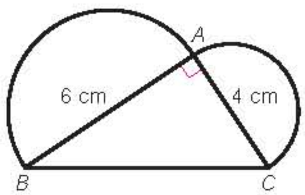
(b)



(c)



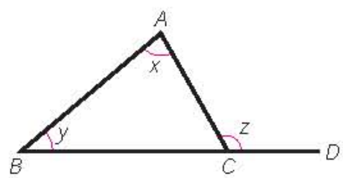
13. In the figure, AB and AC are the diameters of two *semi-circles*, and $AB \perp AC$. Draw the figure in actual size.



Level 2

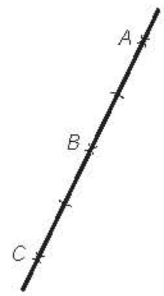
14. **(a)** In ΔABC, BC is produced to point D. Find x, y and z with a protractor.

(b) What is the relation between $x + y$ and z?



semi-circle 半圓

15. In the figure, A, B and C are three points on a straight line, and $AB = BC$.



(a) Draw three parallel lines which pass through A, B and C respectively.

(b) Draw a line which intersects all the lines drawn in **(a)**. Mark the three points of intersections as D, E and F.

(c) Measure DE and EF. Are they equal in length?

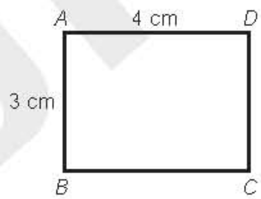
[A copy of the figure is provided in the Appendix.]

16. **(a)** Draw a circle with a radius of 3 cm.

(b) Choose four points A, B, C and D on the *circumference* of **(a)** such that $ABCD$ is a quadrilateral.

(c) Check whether $\angle ACB$ and $\angle ADB$ are equal with a protractor.

17. In the figure, $ABCD$ is a rectangle.



(a) Draw the figure in actual size.

(b) **(i)** If AC is the diameter of a circle, draw the circle in the figure drawn in **(a)**.

(ii) Do the four points A, B, C and D lie on the circumference?

18. **(a)** After running for 10 minutes, find the size of the angle in degrees made by

(i) the minute-hand.

(ii) the hour-hand.

(b) Referring to the figure, find the marked angle between the minute-hand and the hour-hand at 10:10.

19. Figures I and II show a convex pentagon and a concave hexagon respectively.

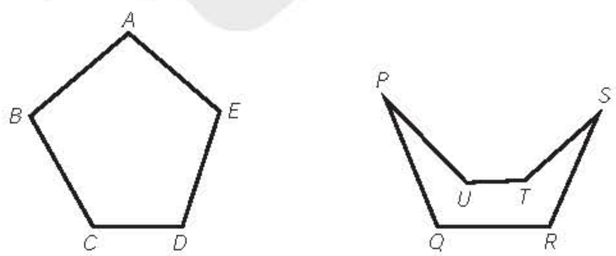


Figure I

Figure II

(a) **(i)** In Figure I, if AB is extended from both ends, will it cut the pentagon?

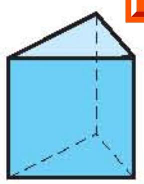
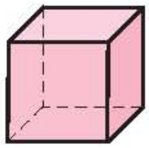
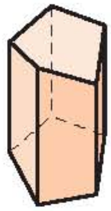
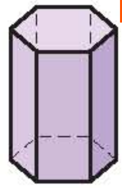
(ii) Extend the remaining sides in the same way, will these lines cut the pentagon?

(b) In Figure II, if PU is extended from both ends, will it cut the hexagon?

(c) From the above results, write down a difference between convex polygons and concave polygons.

circumference 圓周

20. **(a)** Complete the following table.

Prism	 Triangular prism	 Cuboid	 Pentagonal prism	 Hexagonal prism
Number of vertices V				
Number of edges E				
Number of faces F				

(b) An n-prism is a prism with an n-gon as its base.

(i) Express the number of vertices V of an n-prism in terms of n, and explain their relation.

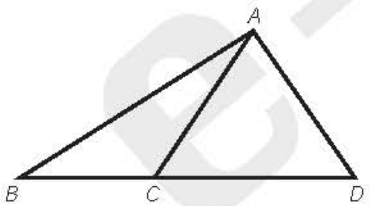
(ii) Express the number of edges E of an n-prism in terms of n, and explain their relation.

(iii) Express the number of faces F of an n-prism in terms of n, and explain their relation.

(c) Determine whether the results in **(b)** satisfy Euler's formula.

MC Question

21. In the figure, BCD is a straight line.



Which of the following angle is an obtuse angle?

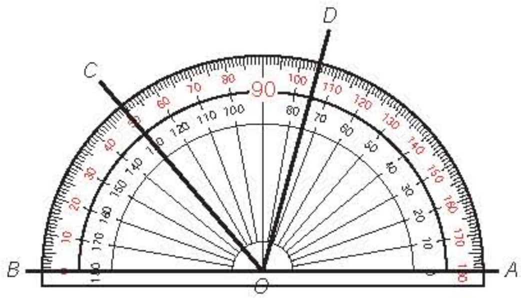
A. $\angle BAC$

B. $\angle BCA$

C. $\angle ABC$

D. $\angle BCD$

22. In the figure, $\angle COD =$



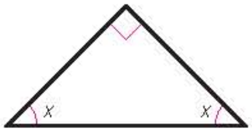
A. 50°.

B. 55°.

C. 60°.

D. 75°.

23. In the figure, find x.



A. 45°

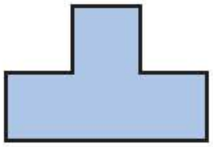
B. 50°

C. 60°

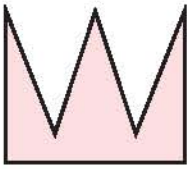
D. 90°

24. Which of the following is a convex polygon?

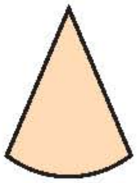
I.



II.



III.



A. I only

B. II only

C. III only

D. None of them

25. At 9:30, the angle between the minute-hand and the hour-hand is

A. 90°.

B. 100°.

C. 105°.

D. 120°.

26. Which of the following are polyhedra?

I. Cube

II. Cuboid

III. Cylinder

A. I and II only

B. I and III only

C. II and III only

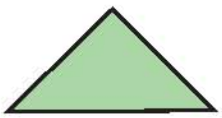
D. I, II and III

Problem-solving and Exploring

Hint for the Title Page Question

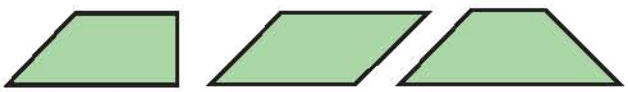
Use a tangram to form the following polygons.

(a) Triangle



(b) Rectangle

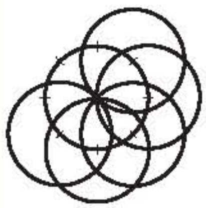
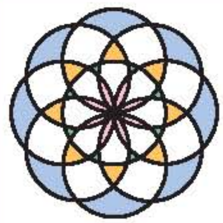
(c) Trapezium



Additional Question

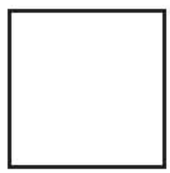
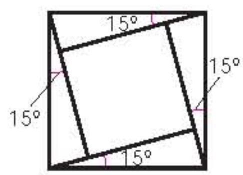
1. We can use different drawing tools to design some beautiful figures.

(a) Here is an example of drawing a figure with a protractor and a pair of compasses.

Step 1	Use the compasses to draw a circle, and keep the distance between the two arms of the compasses for step 3.		Step 2	Overlap the centre of the protractor with the centre of the circle. Then mark a point on the circumference for every 45° to divide the circle into 8 equal parts.	
Step 3	Use the marked points as centres to draw circles with the same radii as in step 1.		Step 4	Colour the figure appropriately.	

Follow the steps above to draw another figure. Set the length of the radii as 6 cm each and divide the circumference into 12 equal parts.

(b) Here is an example of drawing a figure with a protractor and a ruler.

Step 1	Draw a square.	
Step 2	Draw an angle of 15° along each side of the square. A smaller square is formed.	
Step 3	Repeat step 2 in every smaller square drawn to obtain a pattern. Colour the figure appropriately.	

Follow the steps above to draw another figure. Set the sides of the first square as 6 cm each and the angle along each side as 10°.

2. Drawing tools such as rulers, compasses and set squares can also help us to draw the plane figures of some solids. For example, Figure I shows a solid with a uniform cross-section formed by removing a sector from a square. (See Figure II).

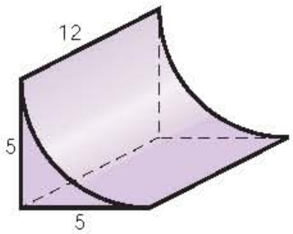


Figure I

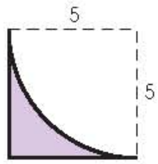
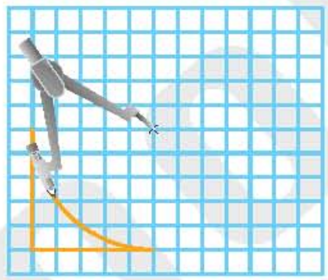
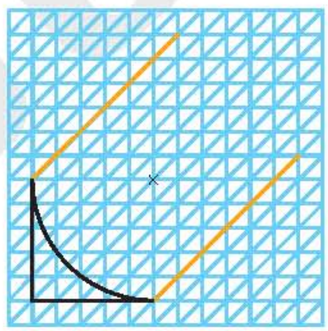
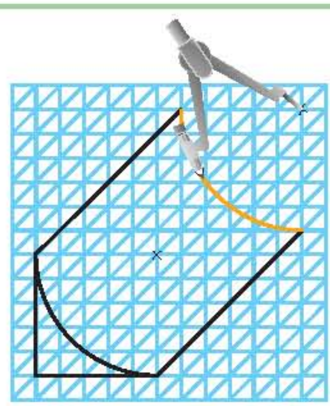


Figure II

The following table shows the steps of drawing the solid.

Step 1	Treat the uniform cross-section as the front surface. Ignore the inclined lines in the grid, draw the front surface first.	
Step 2	Draw the lateral faces along the inclined lines.	
Step 3	Mark the centre of the arc of the back surface, which is 12 units away diagonally from the centre of the arc of the front surface. Then draw another arc.	

Finally, we obtain the completed plane figure as shown in Figure III.

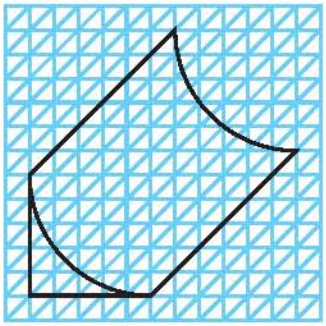


Figure III

Design a solid and draw it on an oblique grid with the drawing tools introduced in this chapter.

Appreciating Mathematics
Enrichment

Interesting Polyhedra

A Archimedean solids

Observe the polyhedra in the following figures.

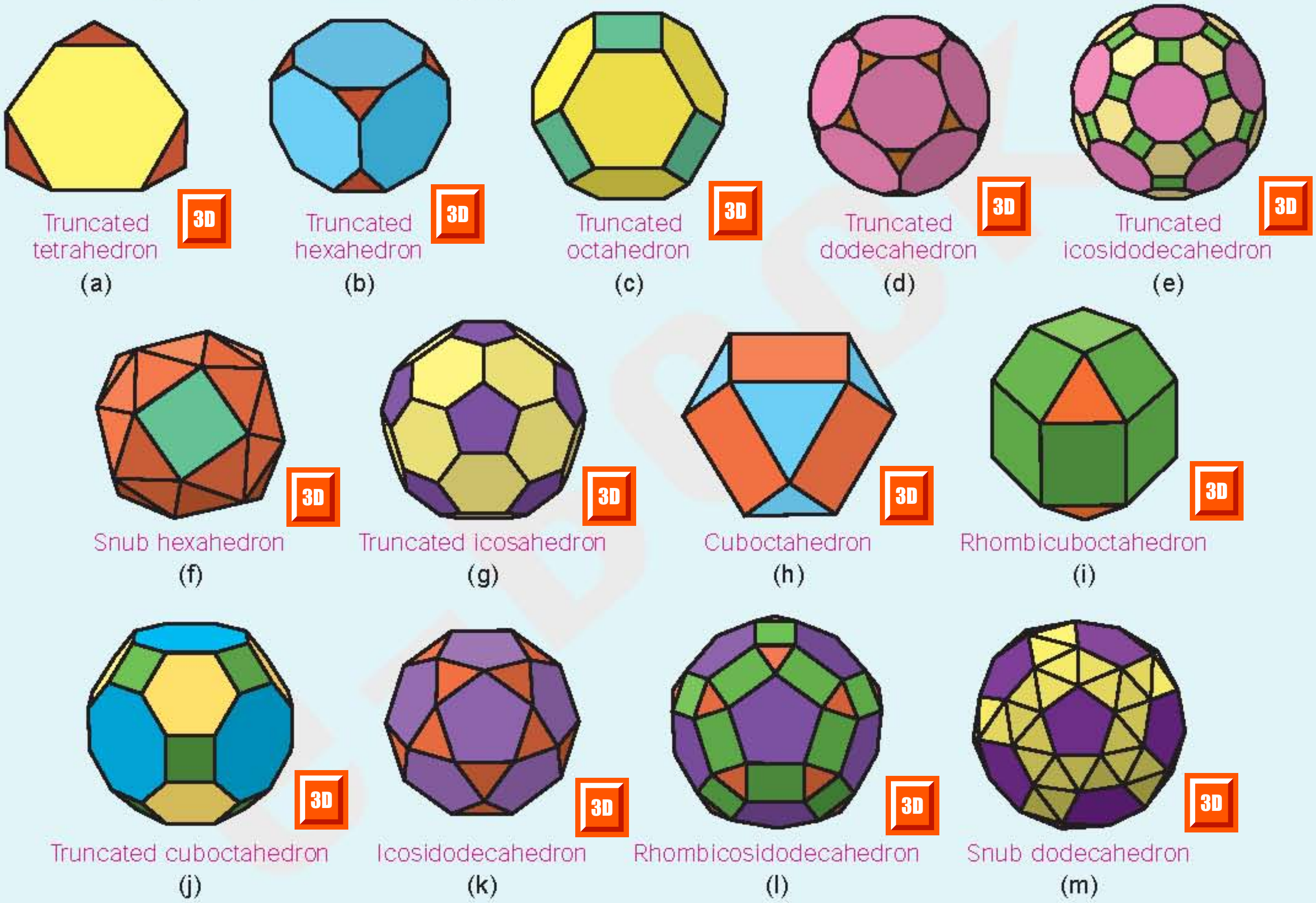


Figure 3.31

Have you noticed that each polyhedron above has more than one type of regular polygon as its faces? For example, the polyhedron in Figure 3.31(j) is made up of squares, regular hexagons and regular octagons.

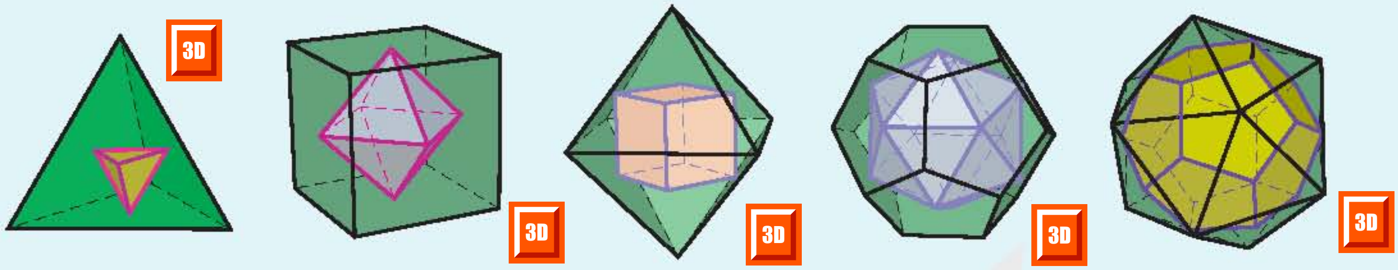
Archimedes (287BC - 212BC), a great Greek mathematician, studied these 13 polyhedra. In memory of his contribution, these polyhedra are named after him as **Archimedean solids**.

The nets of different Archimedean solids are provided in the Student's CD for model making.

truncated tetrahedron 截角正四面體
truncated octahedron 截角正八面體
truncated icosidodecahedron 大六十二面體
truncated icosahedron 截角正二十面體
rhombicuboctahedron 小二十六面體
icosidodecahedron 截盡角正十二面體
snub dodecahedron 獅鼻正十二面體
truncated hexahedron 截角正六面體
truncated dodecahedron 截角正十二面體
snub hexahedron 獅鼻正六面體
cuboctahedron 截盡角正八面體
truncated cuboctahedron 大二十六面體
rhombicosidodecahedron 小六十二面體
Archimedean solid 阿基米德立體

B Dual polyhedra

For some regular polyhedra, if they fit together, one regular polyhedron can be put inside another such that all the vertices of the former touch the centres of the faces of the latter. (See Figure 3.32)



Two regular tetrahedra (a) | A cube (outside) and a regular octahedron (inside) (b) | A regular octahedron (outside) and a cube (inside) (c) | A regular dodecahedron (outside) and a regular icosahedron (inside) (d) | A regular icosahedron (outside) and a regular dodecahedron (inside) (e)

Figure 3.32

Observe Figure 3.32(b). Since the vertices of the regular octahedron are at the centres of the faces of the cube, the number of vertices of the regular octahedron is the same as the number of faces of the cube.

If two regular polyhedra satisfy the following conditions:

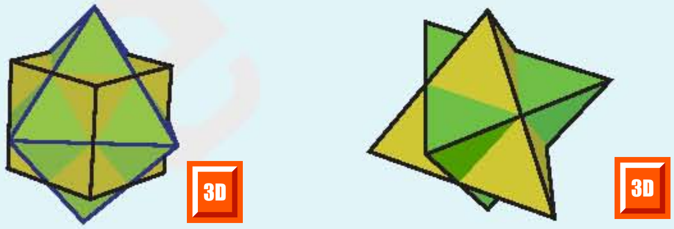
1. the number of their edges is the same;

2. the number of faces of one of them is the same as the number of vertices of the other,

then these two polyhedra are called **dual polyhedra**.

Thus, all the pairs of polyhedra in Figure 3.32 are dual polyhedra.

Furthermore, if two dual polyhedra fit together, we can put them in a way such that all their intersecting edges are perpendicular to each other. (See Figure 3.33)



A cube and a regular octahedron (a) | Two regular tetrahedra (b)

Figure 3.33

Different dual polyhedra are provided in the Student's CD for further study.

dual polyhedra 對偶多面體